Large-scale multidisciplinary design optimization under uncertainty using graph-based modeling

Wind Energy Systems Engineering (WESE) workshop

DTU, Roskilde, Denmark

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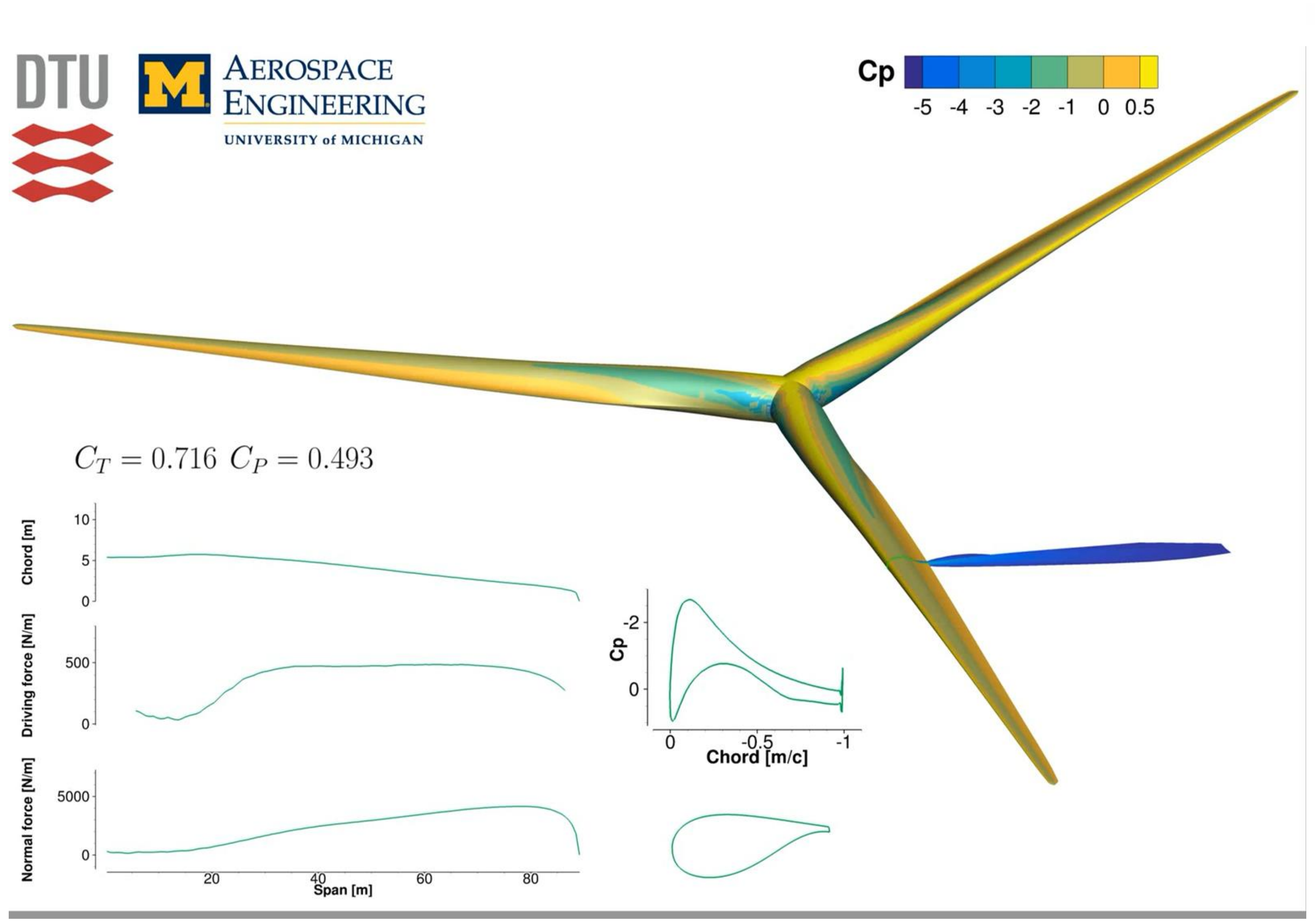
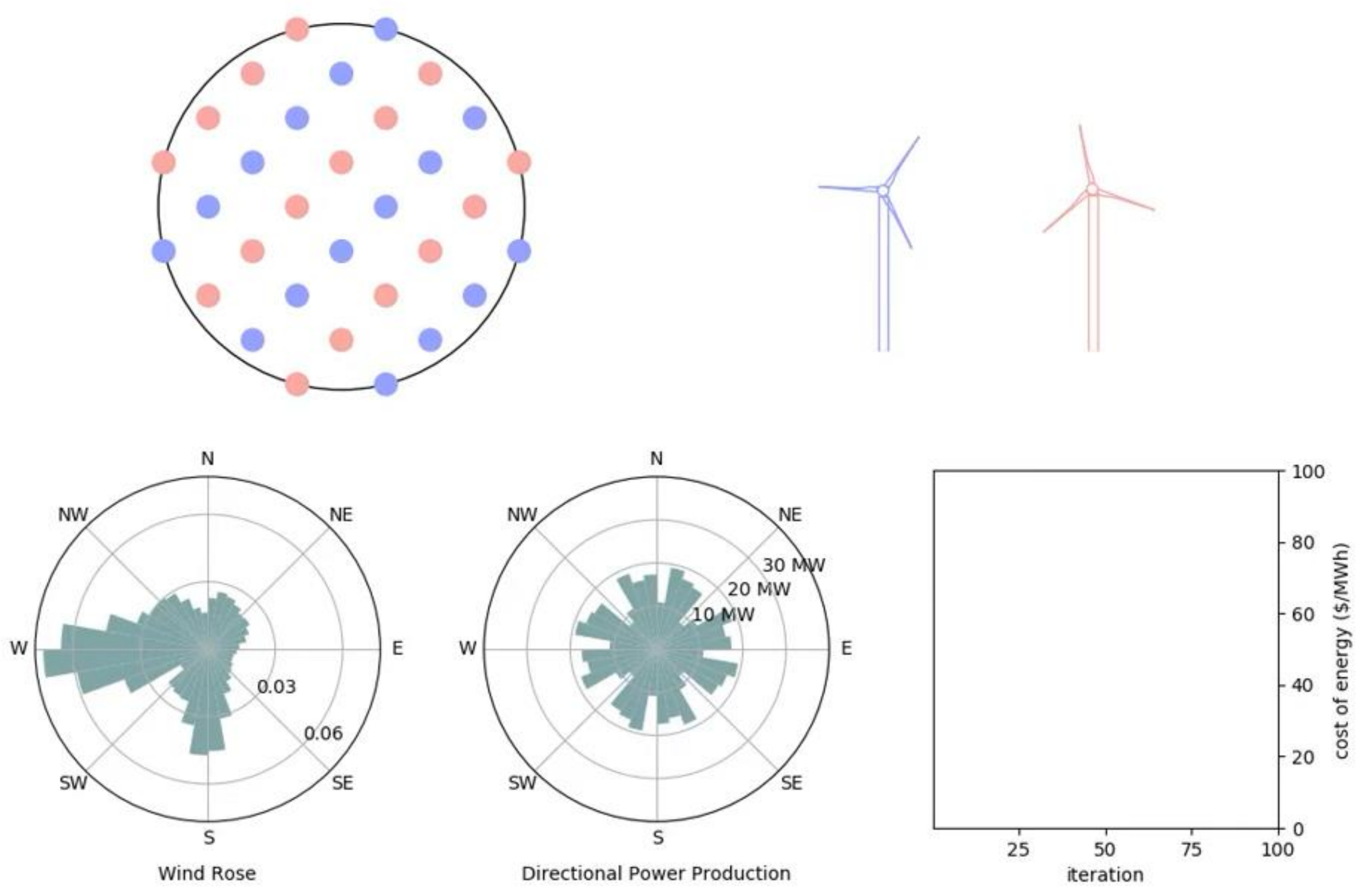
Design optimization in wind energy system design



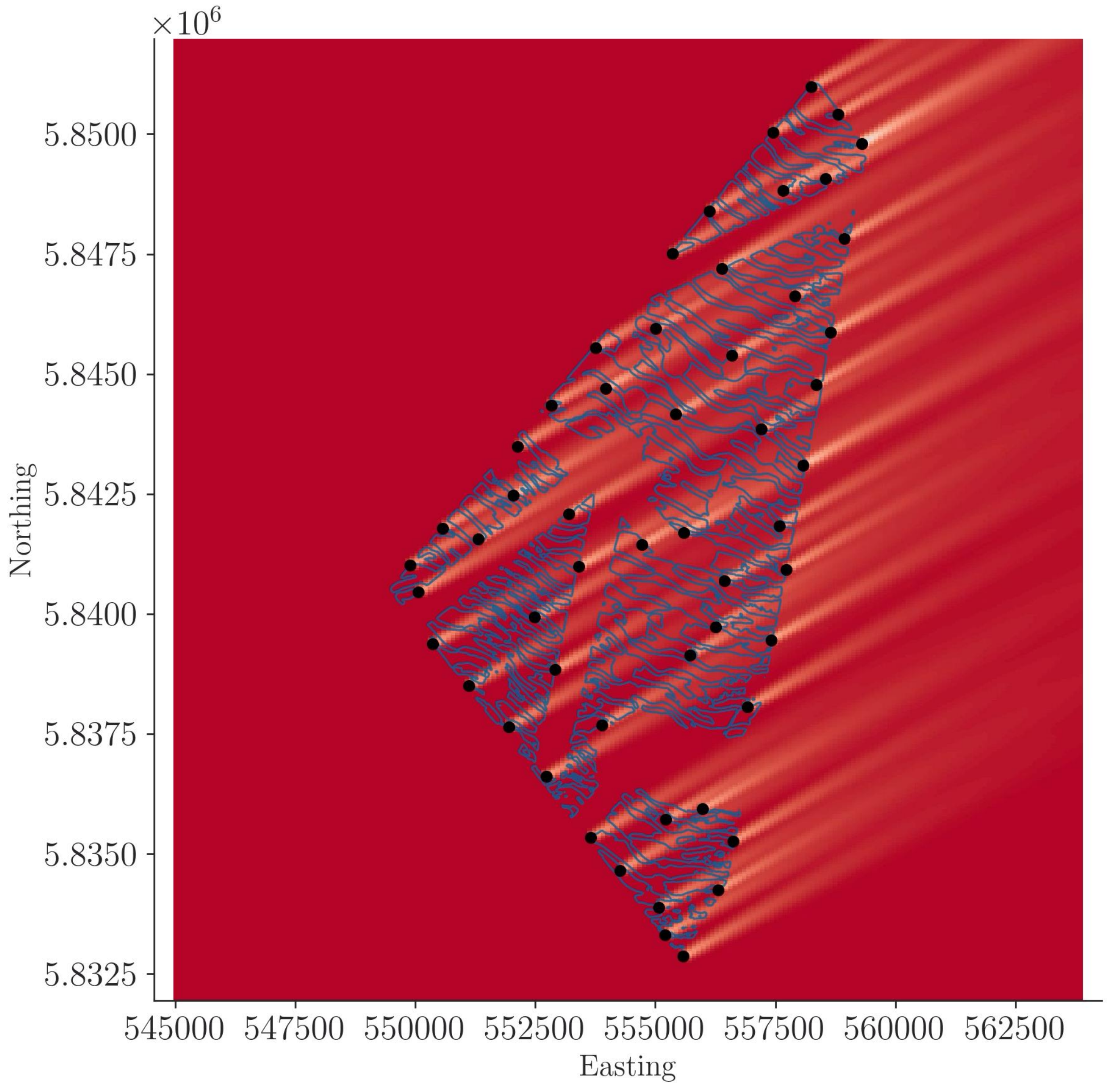
Wind turbine design

Madsen, M. H. A., Zahle, F., Sørensen, N. N., & Martins, J. R. (2019). Multipoint high-fidelity CFD-based aerodynamic shape optimization of a 10 MW wind turbine. *Wind Energy Science*, *4*(2), 163-192. https://www.youtube.com/watch?v=5gSY7IbUuMA



Turbine-tower-layout design

Stanley, A. P., & Ning, A. (2019). Coupled wind turbine design and layout optimization with nonhomogeneous wind turbines. Wind Energy Science, 4(1), 99-114. https://flow.byu.edu/posts/coupled-turbine-farm



Wind farm design

Dunn, R. C., Joshy, A. J., Lin, J. T., Girerd, C., Morimoto, T. K., & Hwang, J. T. (2023). Scalable enforcement of geometric non-interference constraints for gradient-based optimization. *Optimization and Engineering*, 1-34.

Design optimization

Use of numerical optimization as a tool to assist in engineering design

objective function

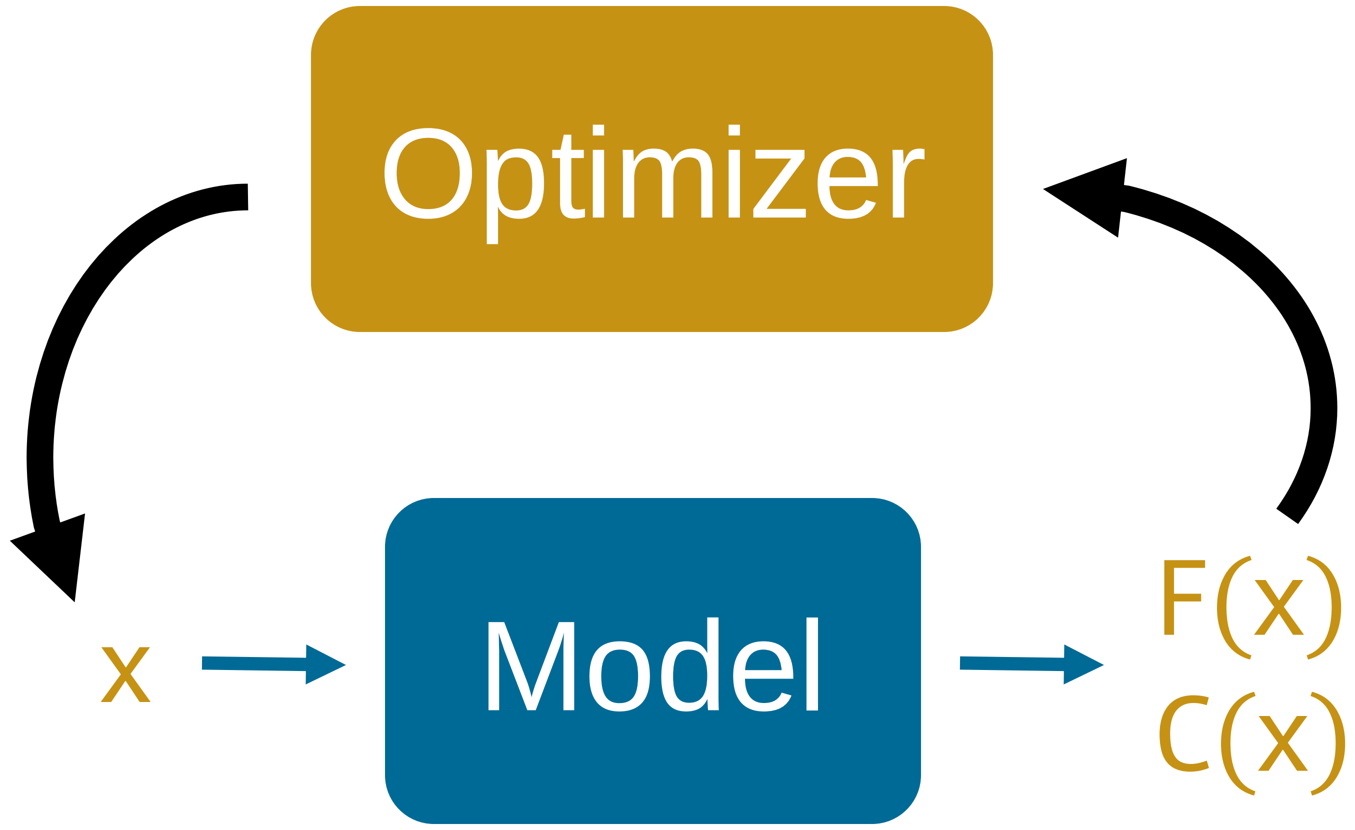
$$\underset{x \in \mathbb{R}^n}{\text{minimize}} \quad F(x)$$

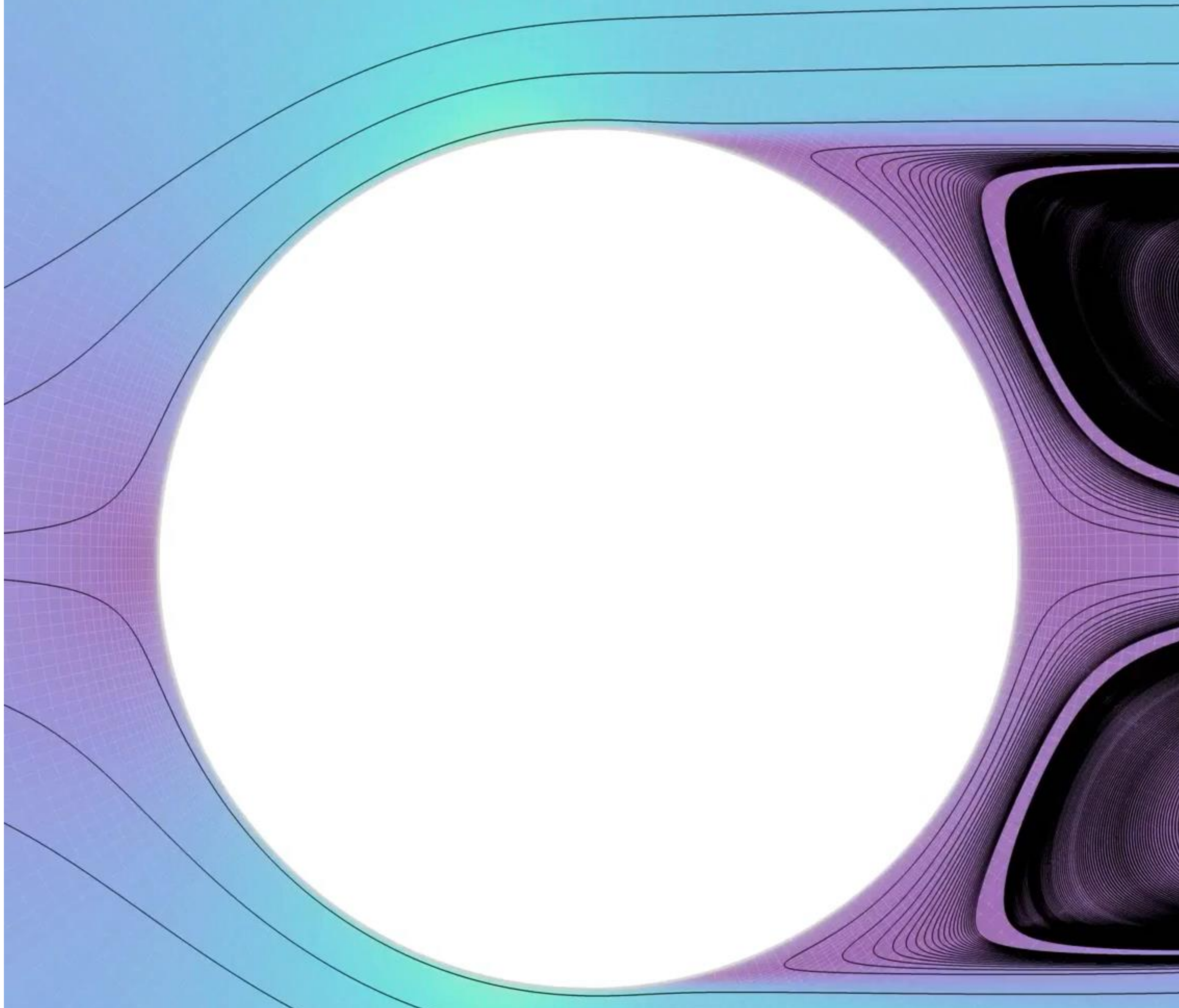
$$\text{subject to} \quad C_i(x) \leq 0, \forall i \in \{1, \dots, m\}$$

$$l \leq x \leq u$$

design variables

constraint functions



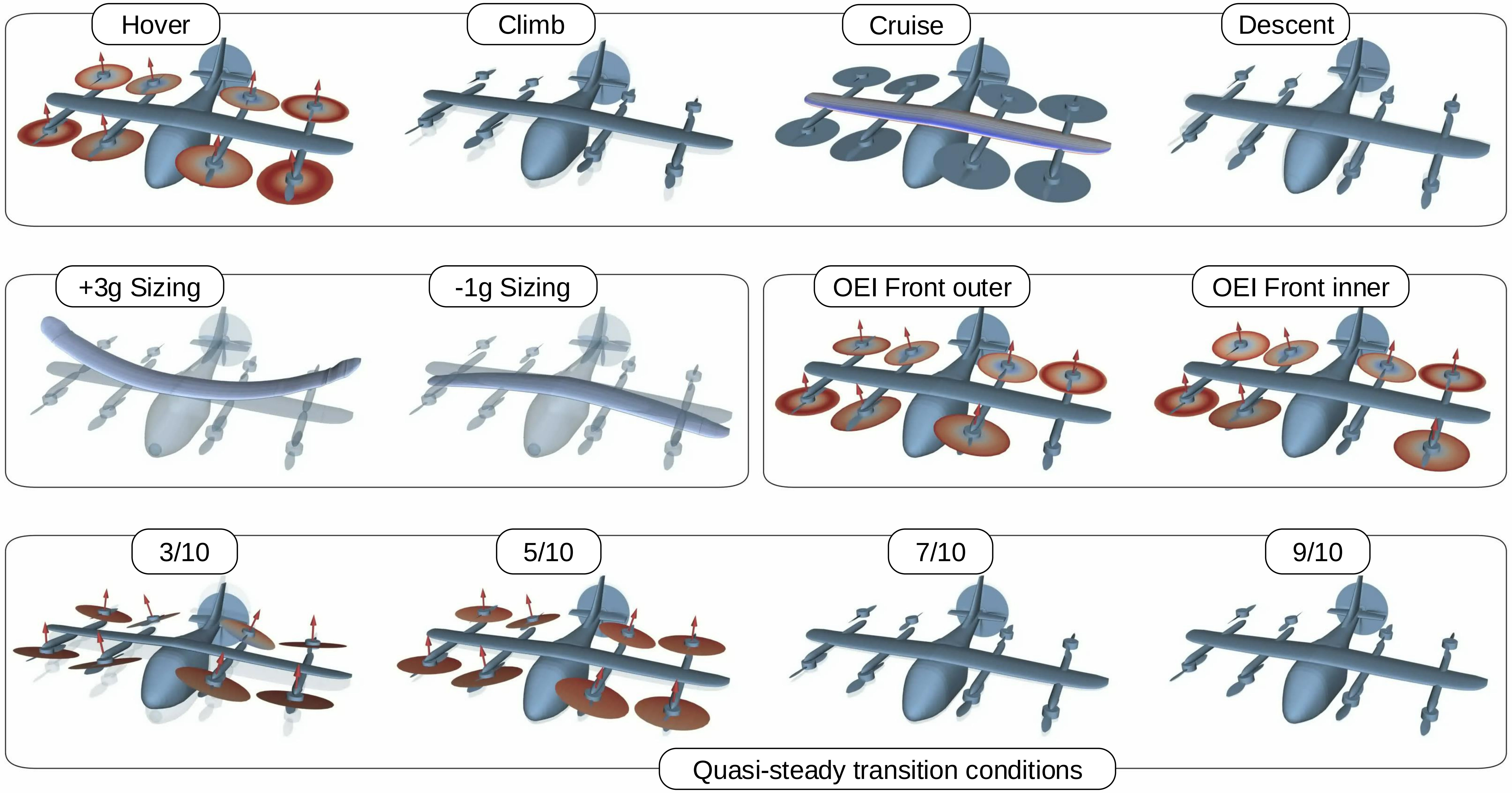


He, X., Li, J., Mader, C. A., Yildirim, A., & Martins, J. R. (2019). Robust aerodynamic shape optimization—from a circle to an airfoil. Aerospace Science and Technology, 87, 48-61. https://www.youtube.com/watch?v=FHYTBguMfWc

Large-scale multidisciplinary design optimization (MDO)

Dozens or more design variables

Complex models considering multiple disciplines, subsystems, and conditions



Objective F(x)	**Gross weight**	
Design variables x	Rotor radii	8
	Blades' twist, chord profiles	72
	Motors' dimensions	18
	Wing/tail dimensions	15
	Battery mass	1
	Rotor speeds	72
	Rotor tilt angles	124
	Vehicle trim variables	26
	Wing structural sizing	40
	Total design variables	**376**
Constraints C(x)	Geometric constraints	46
	Aircraft equations of motion	18
	Structural constraints	163
	Propulsion constraints	6
	Acoustic constraints	5
	Total constraints	**238**

Ruh, M. L., Sarojini, D., Fletcher, A., Asher, I., & Hwang, J. T. (2023). Large-scale multidisciplinary design optimization of the NASA lift-plus-cruise concept using a novel aircraft design framework. arXiv preprint arXiv:2304.14889.

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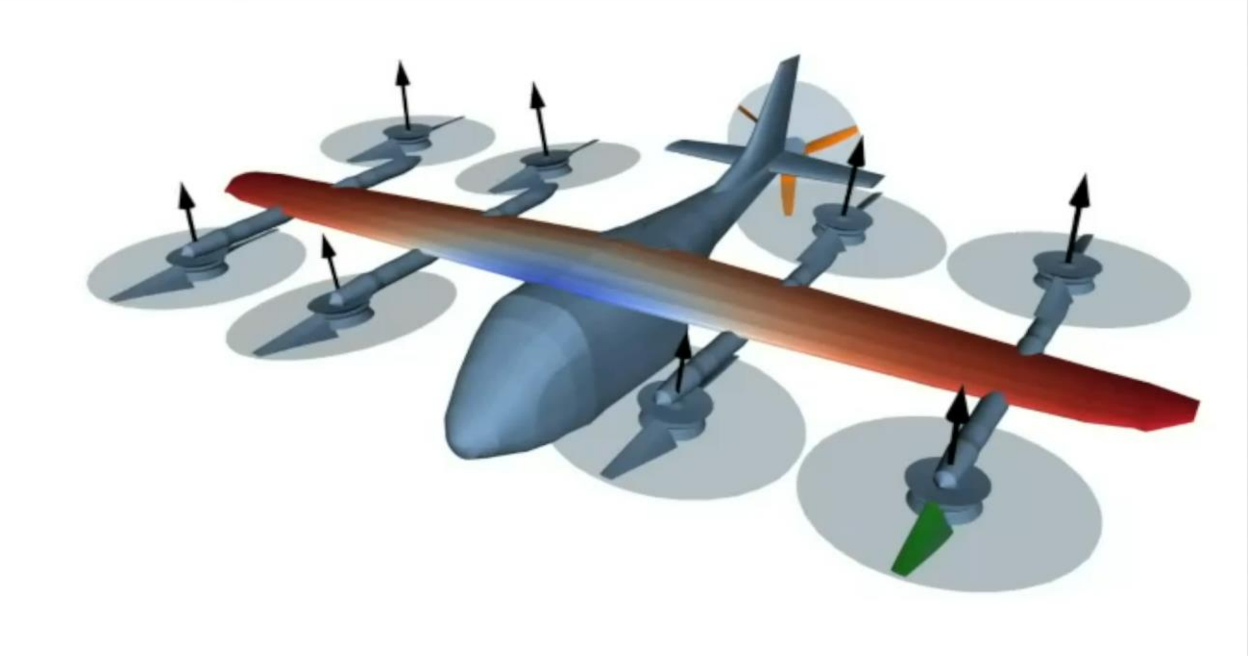
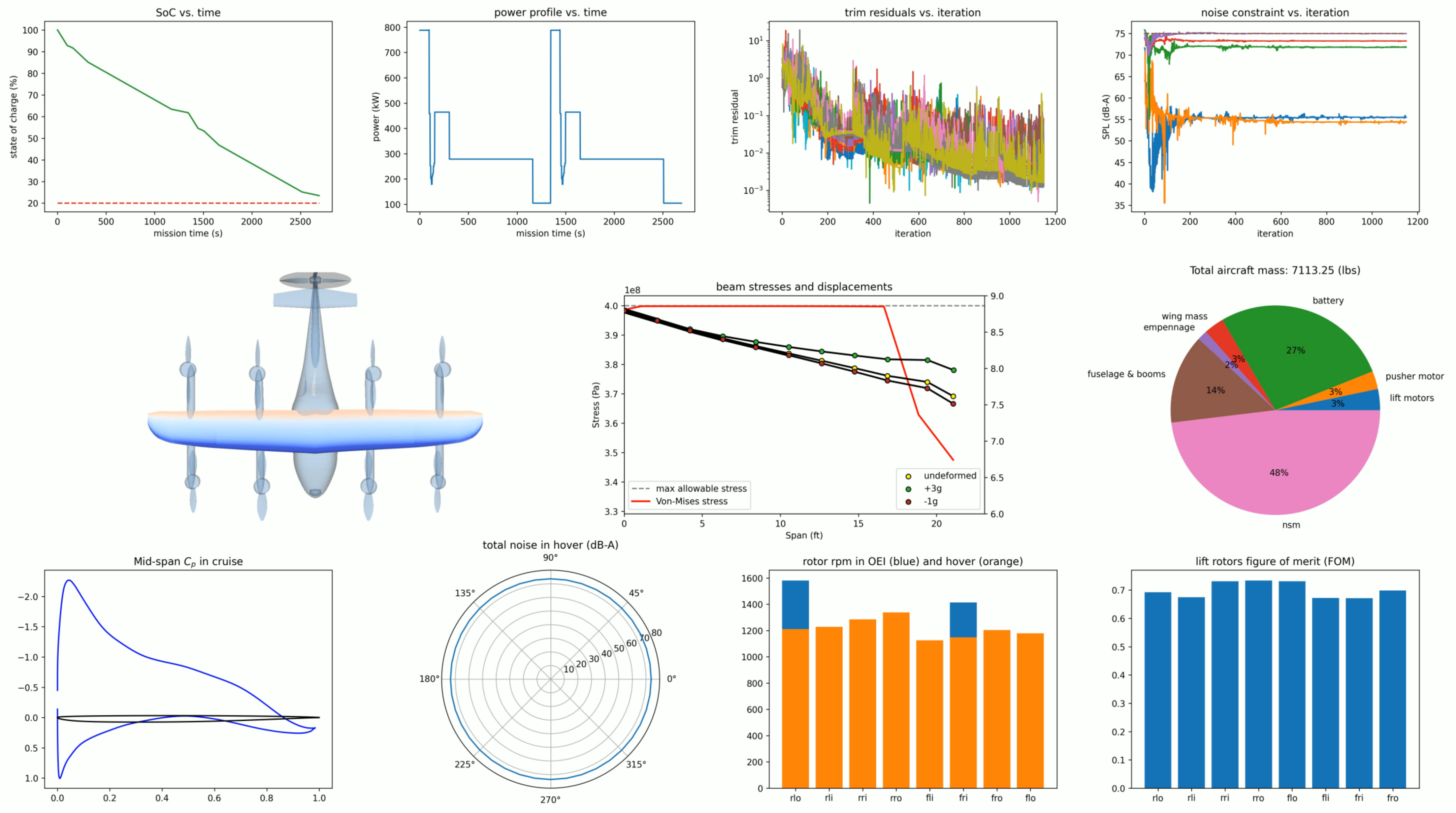
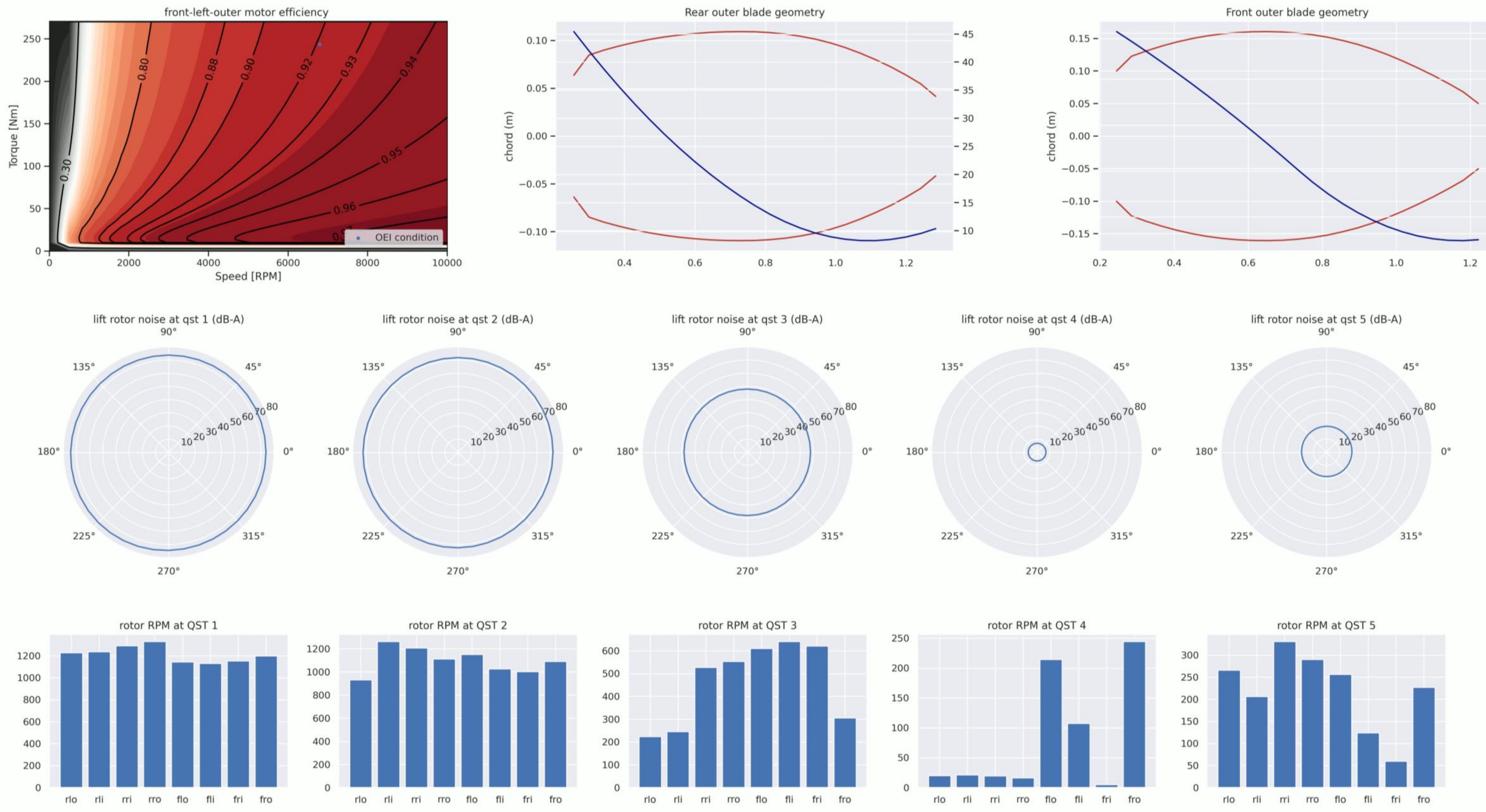






Aerodynamics (wing) | Battery | Aerodynamics (rotor) | Weights | Motor | Acoustics

Large-scale multidisciplinary design optimization (MDO) under uncertainty using graph-based modeling

Background—classical methods for large-scale MDO

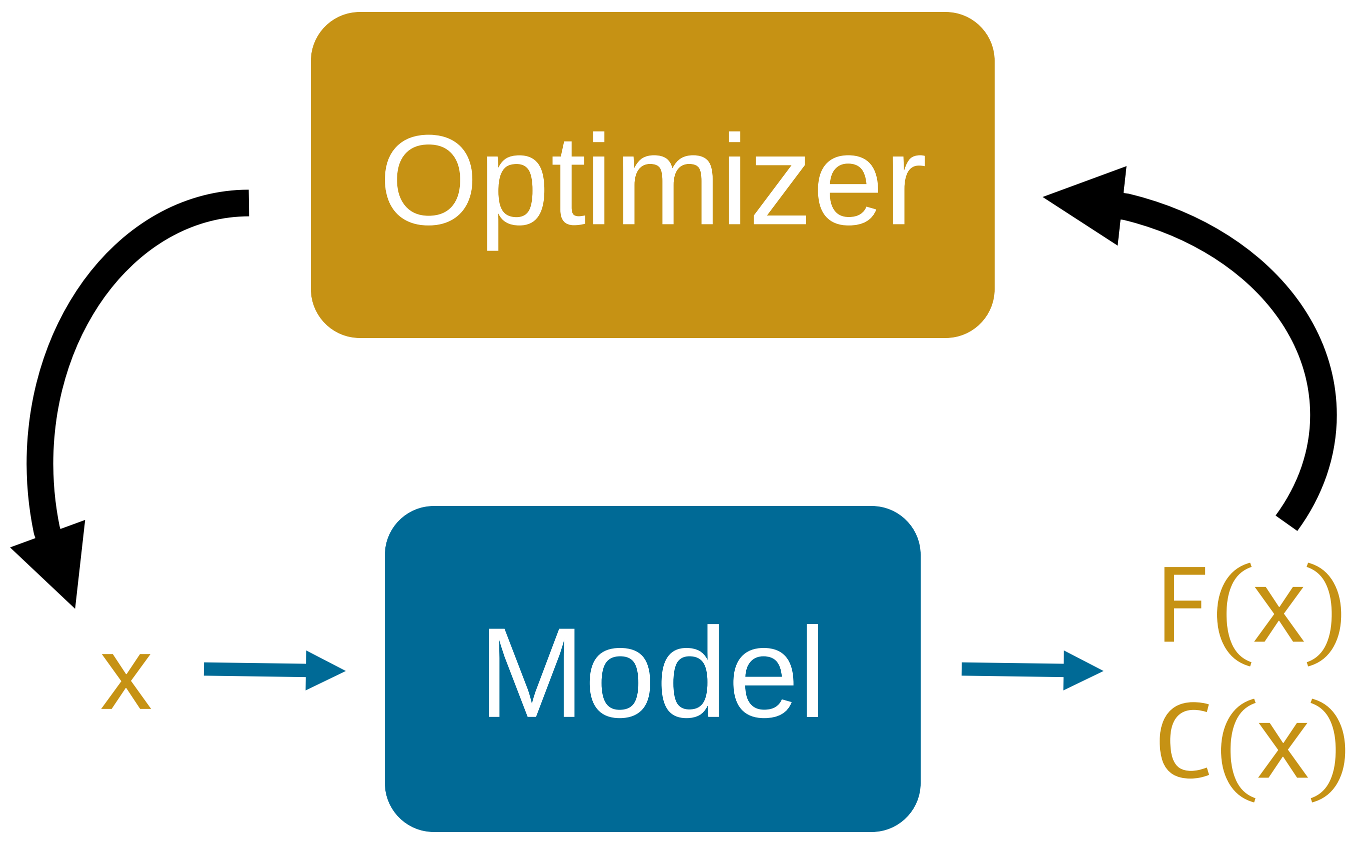
Graph-based modeling for large-scale MDO

Application: wind farm layout design optimization

Towards large-scale MDO under uncertainty

Large-scale MDO algorithms use either gradient-based or gradient-free optimizers

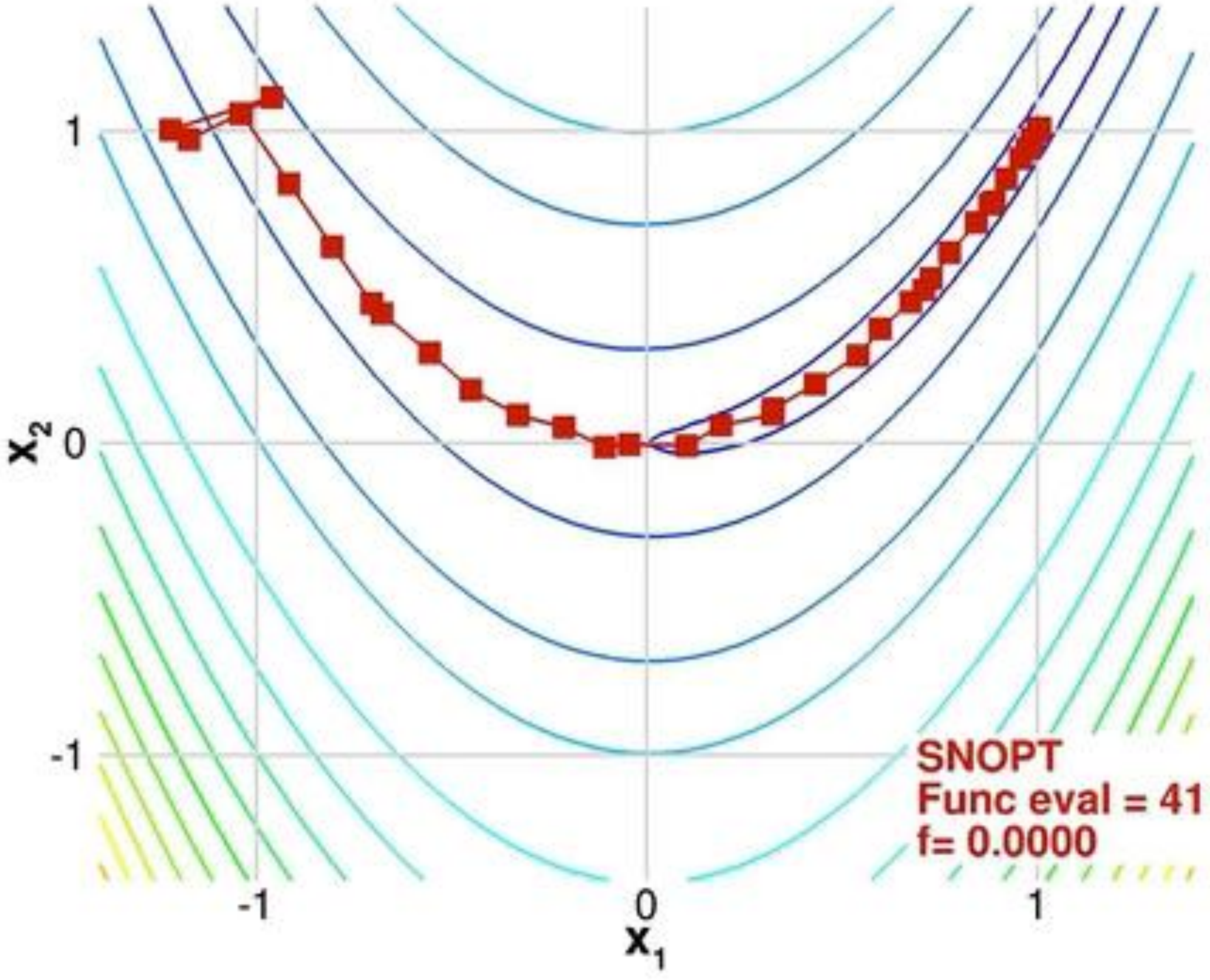
General-purpose
optimization
algorithm



Gradient-based optimizer

(SNOPT)

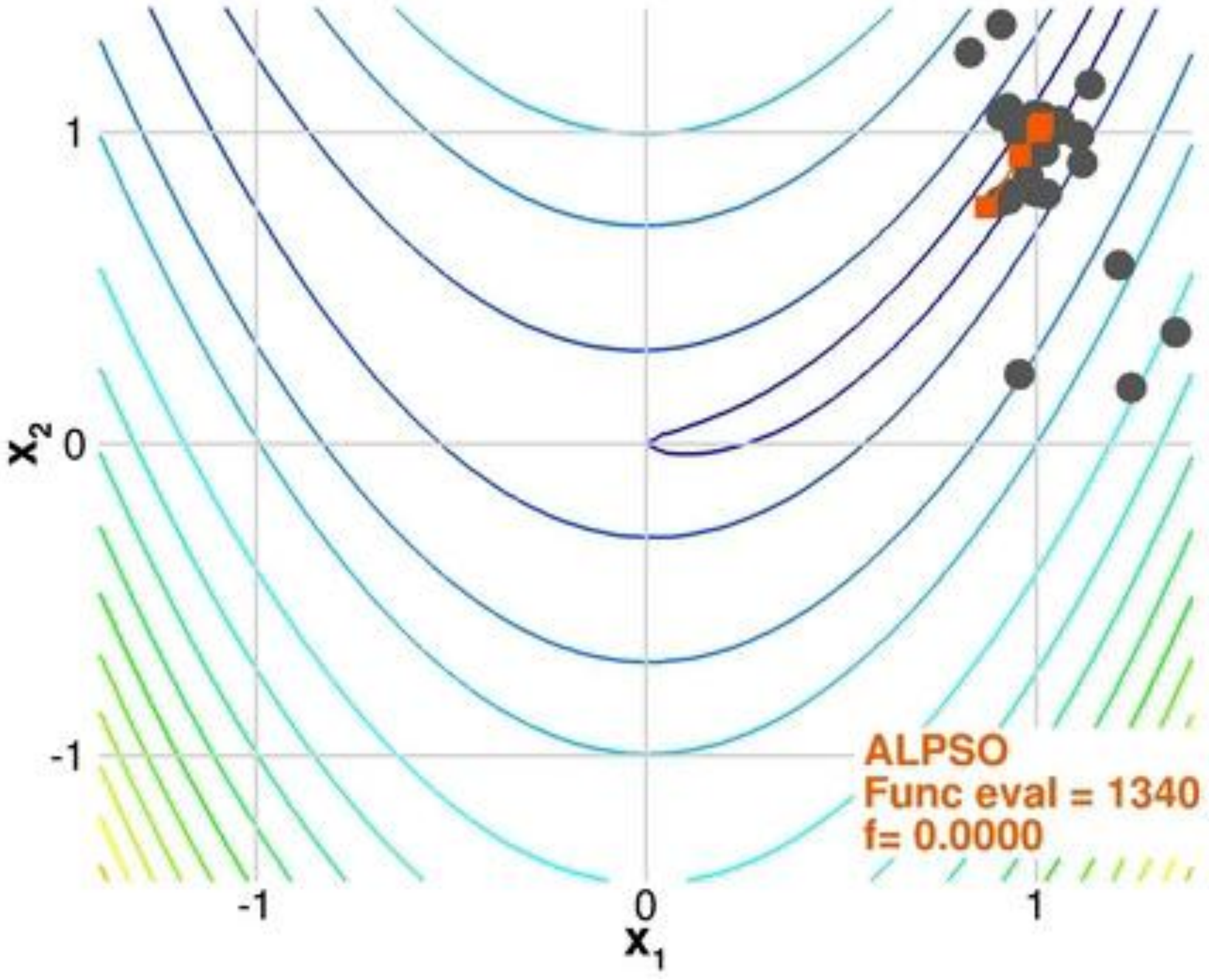
41 iterations



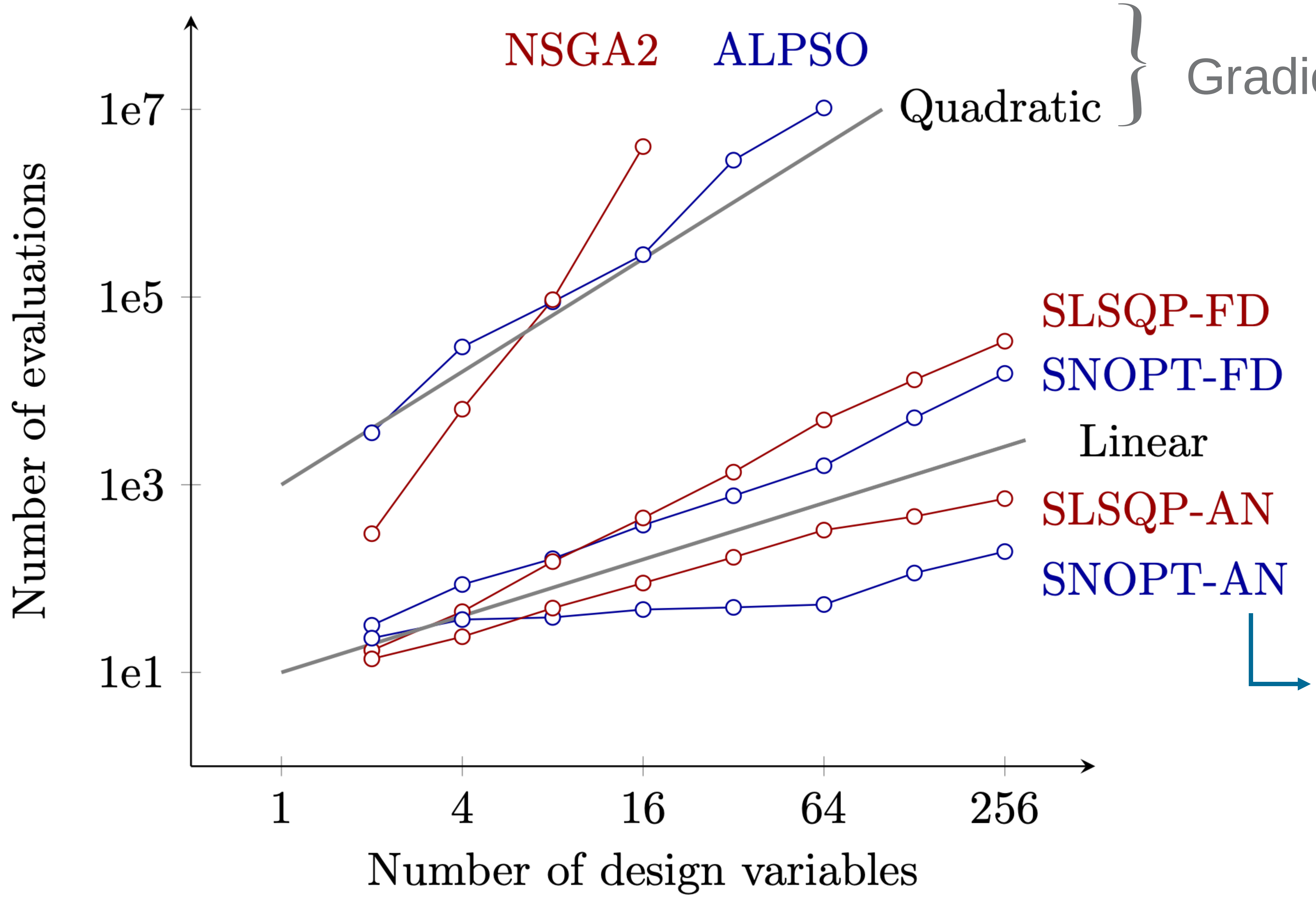
Gradient-free optimizer

(ALPSO)

1340 iterations



Large-scale MDO requires gradient-based optimization and adjoint sensitivity analysis



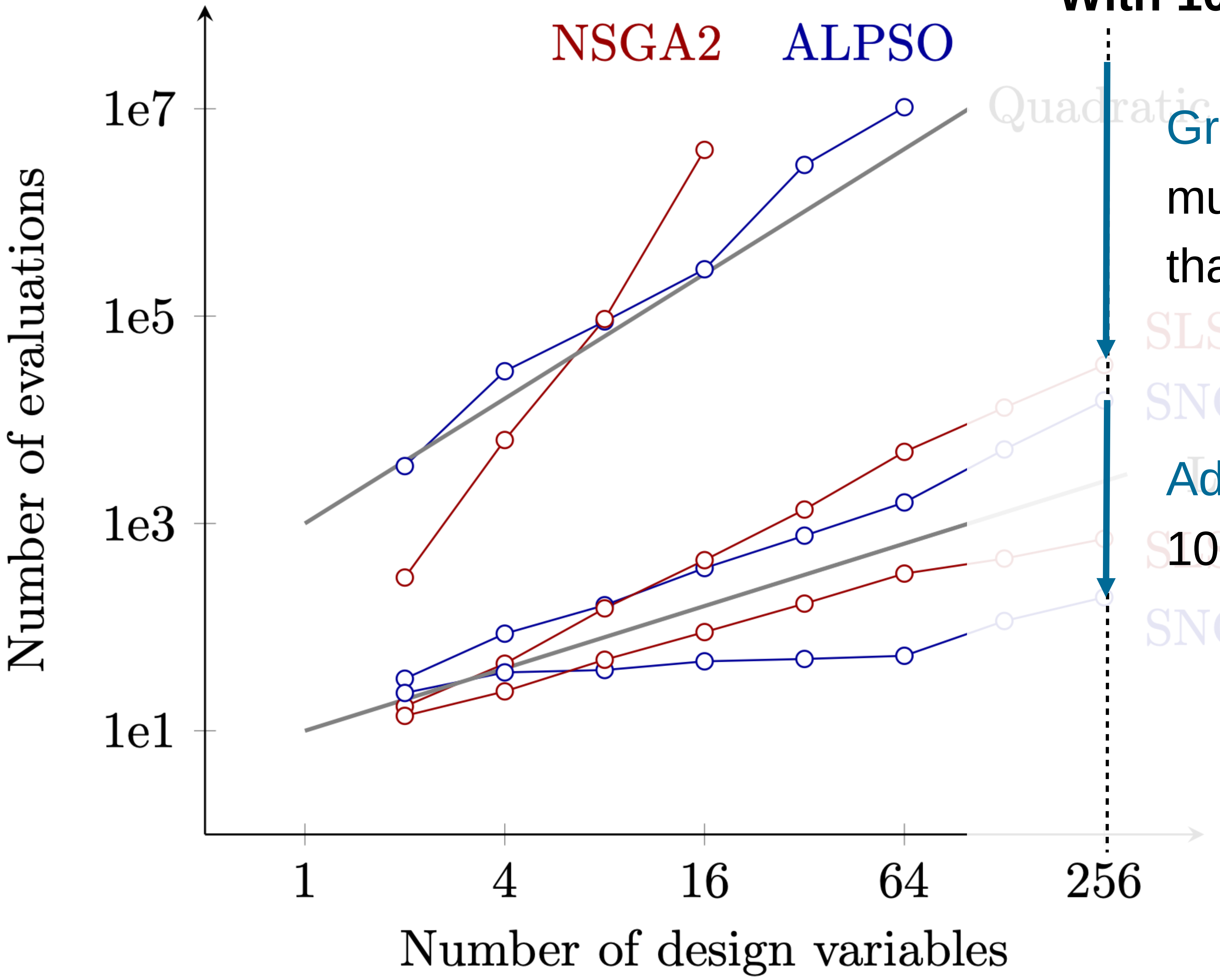
Gradient-free optimizers

Gradient-based optimizers

AN: Adjoint sensitivity analysis (for gradient computation)

[Hwang, 2015, University of Michigan]

Large-scale MDO requires gradient-based optimization and adjoint sensitivity analysis



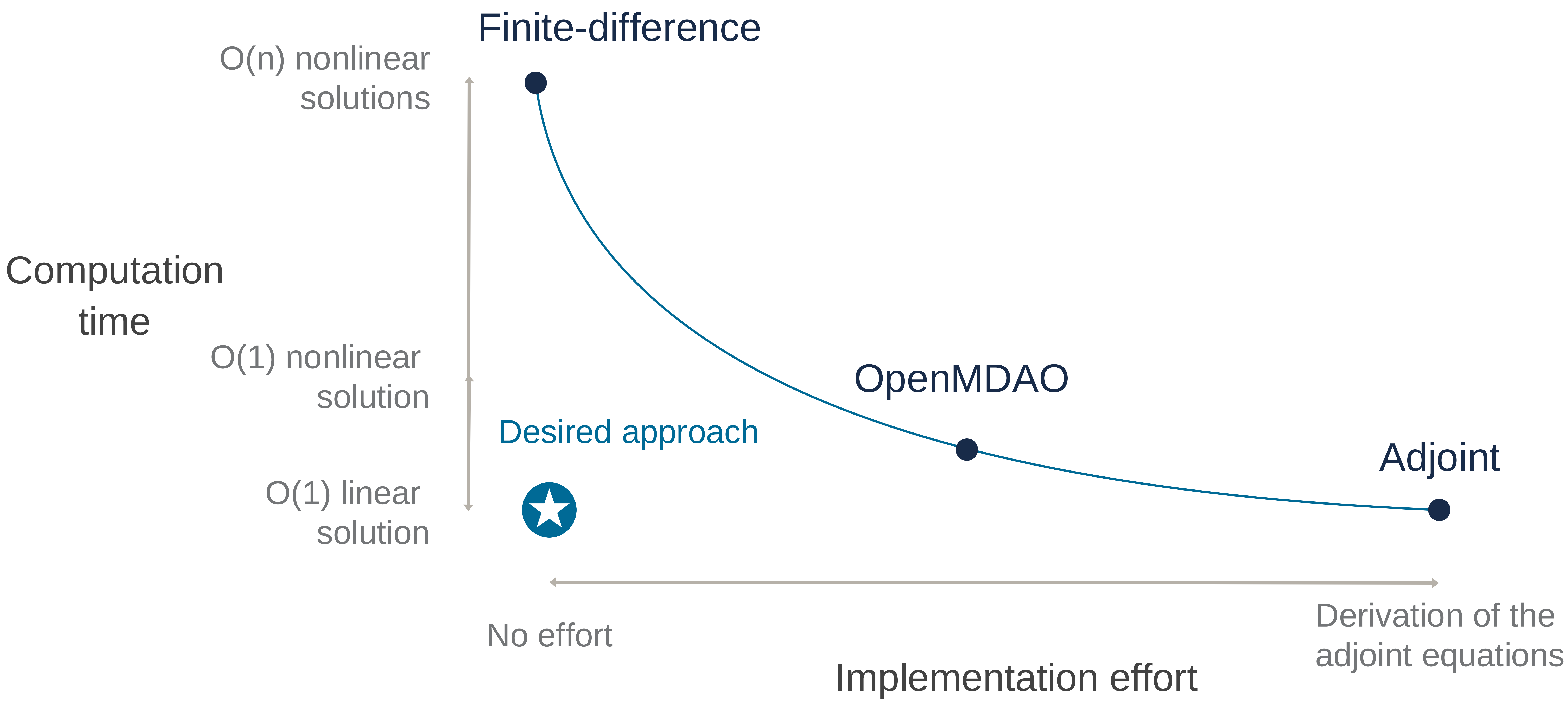
With 100s of design variables:

Gradient-based optimization is multiple orders of magnitude faster than gradient-free optimization

Adjoint sensitivity analysis computes the gradient 100x faster than finite-difference approximations

[Hwang, 2015, University of Michigan]

We desire a sensitivity analysis method with both low computational time & low implementation effort



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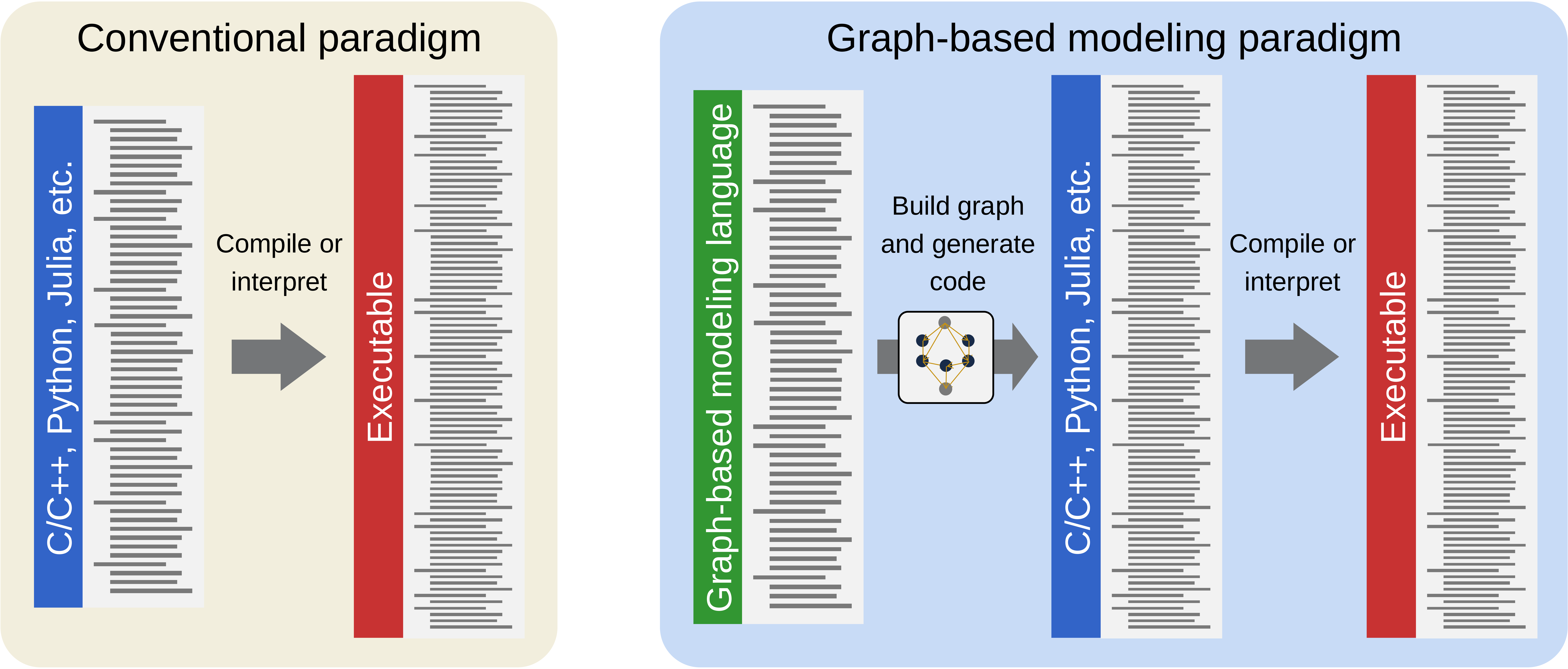
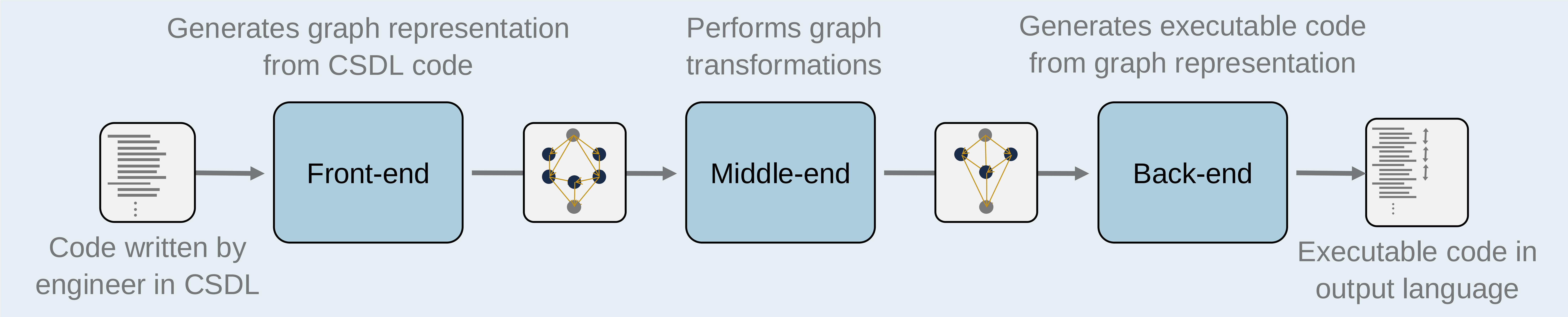
Graph-based modeling is a new paradigm for constructing and working with models



Graph-based modeling enables automation of multidisciplinary adjoint sensitivity analysis



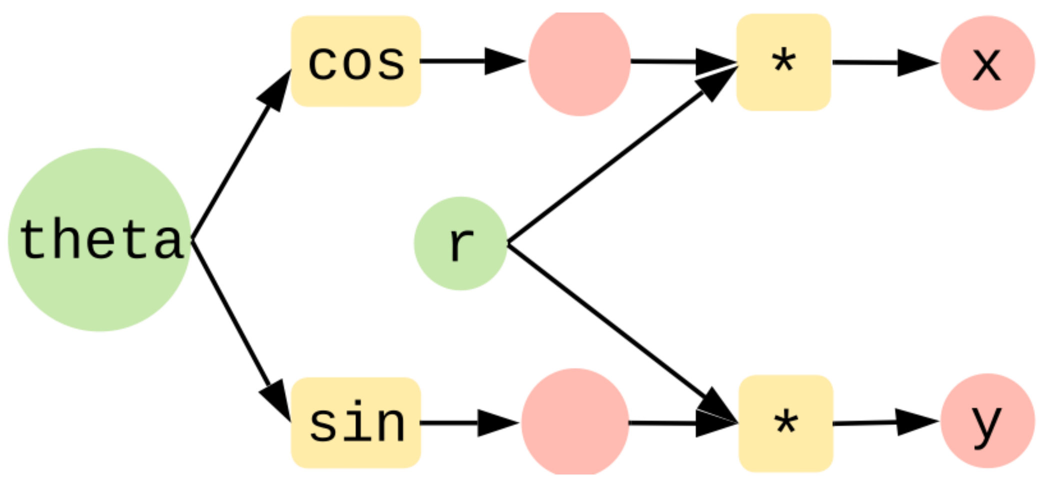
CSDL is a new algebraic modeling language

```
r = declare_variable('r')
theta = declare_variable('theta')

x = r * cos(theta)
y = r * sin(theta)

register_output('x', x)
register_output('y', y)
```

Model represented as a directed acyclic graph



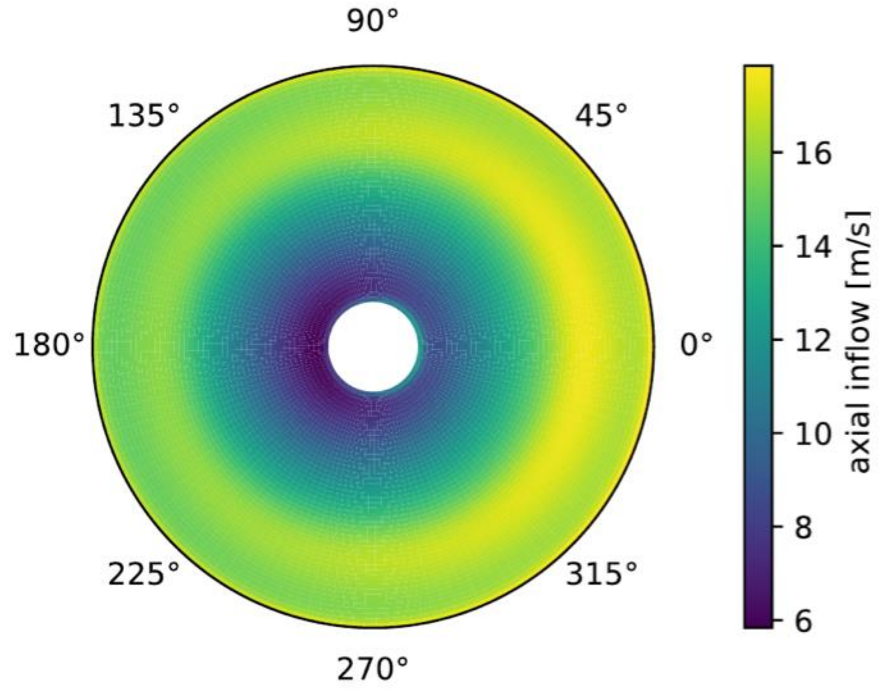
Generated code performs automatic sensitivity analysis

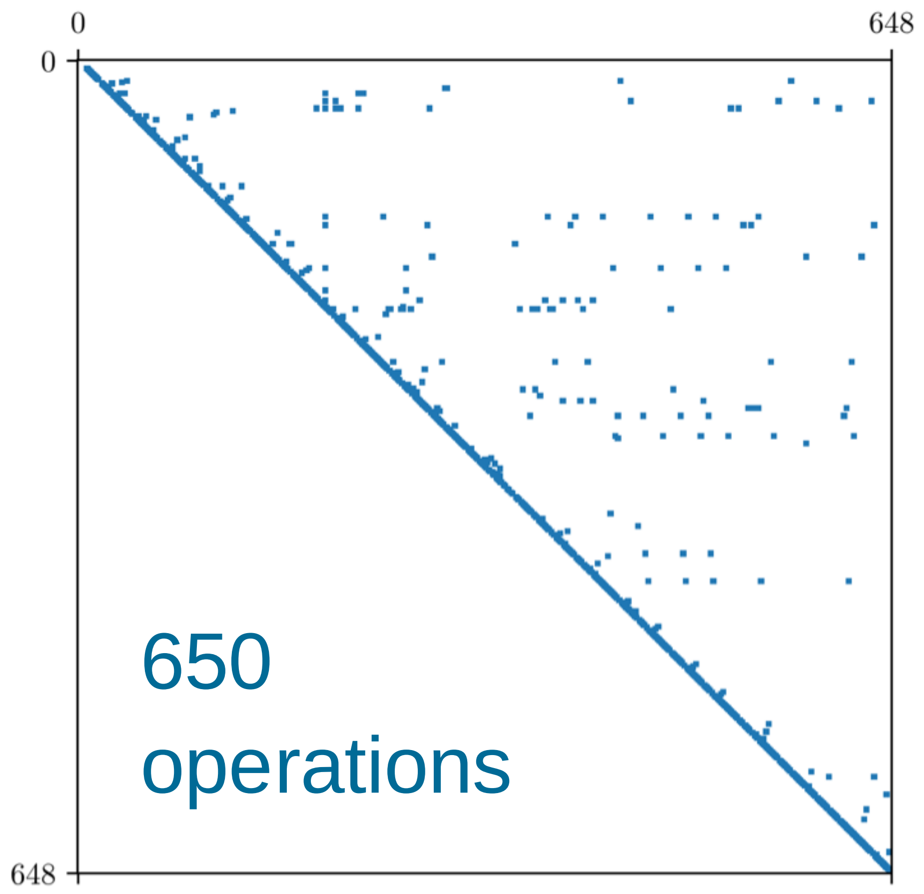
Machine-generated code

Gandarillas, V., Joshy, A. J., Sperry, M. Z., Ivanov, A. K., & Hwang, J. T. (2024). A graph-based methodology for constructing computational models that automates adjoint-based sensitivity analysis. Structural and Multidisciplinary Optimization, 67(5), 76.

Computational graphs of real-world models

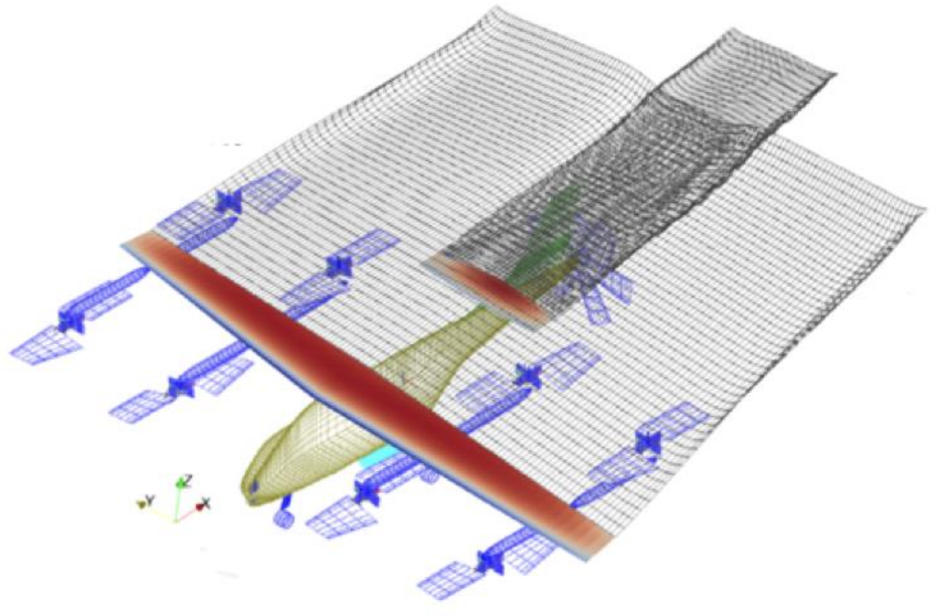
Pitt—Peters model

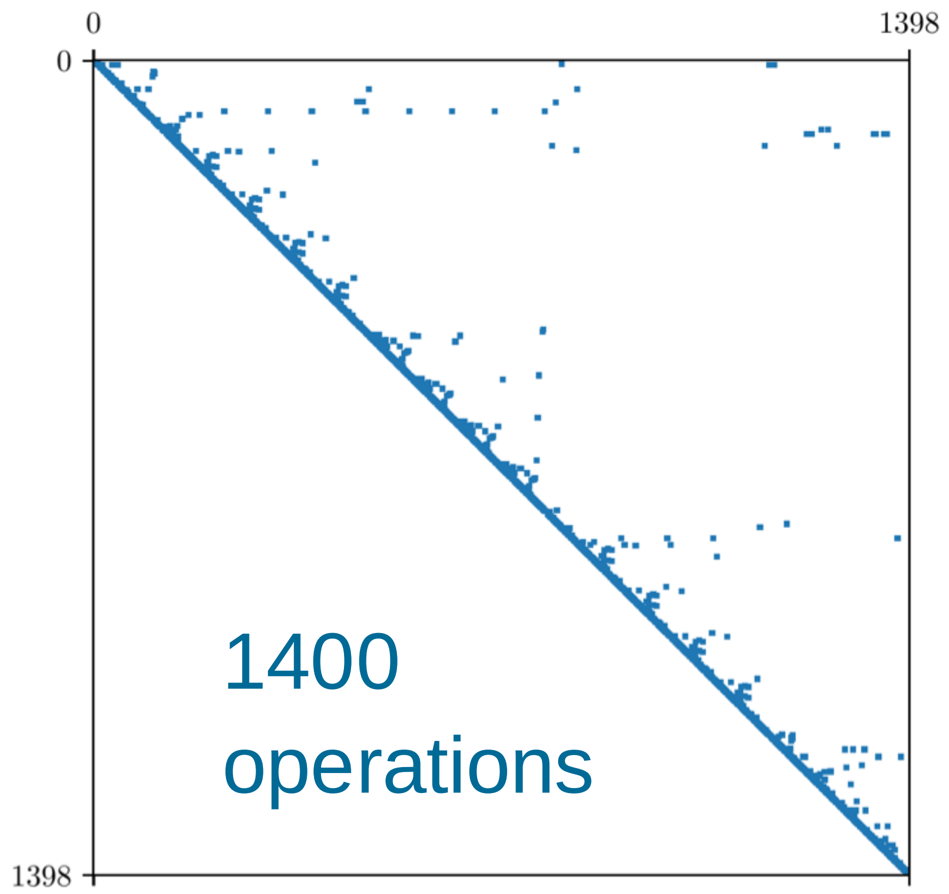




650
operations

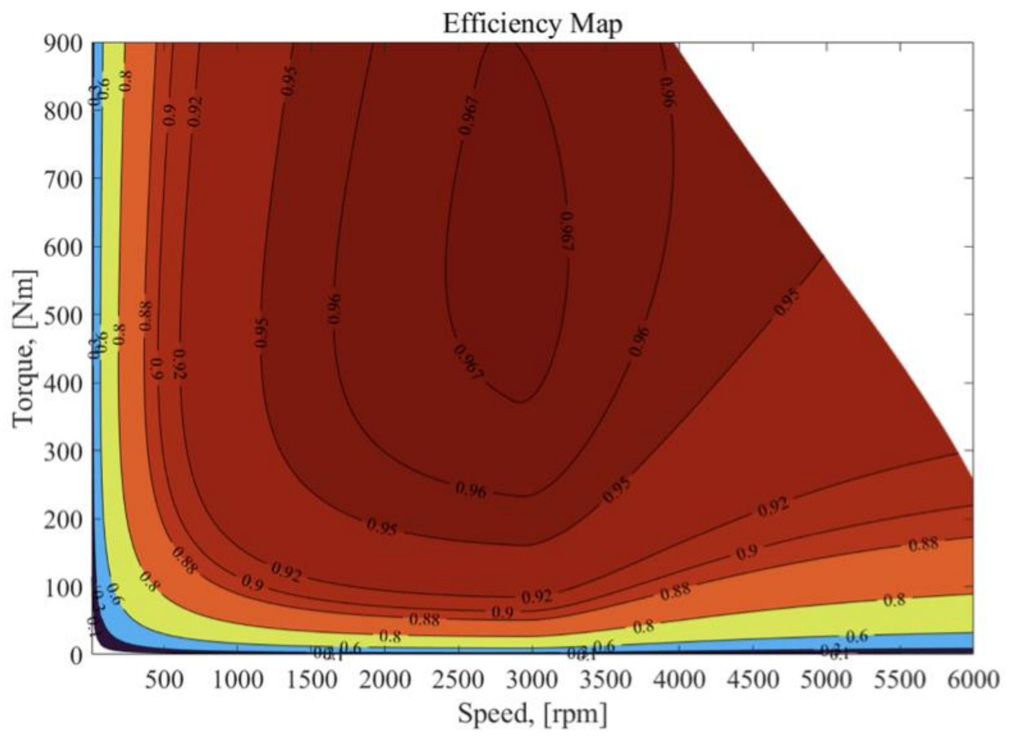
Free-wake model

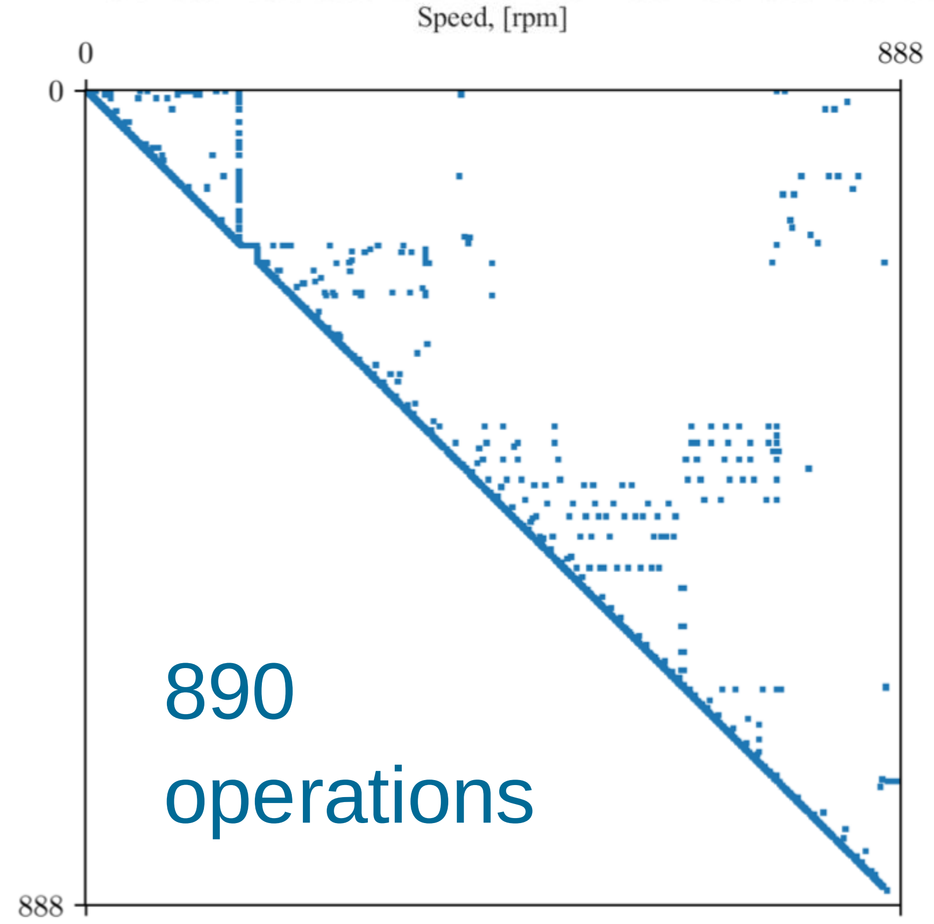




1400
operations

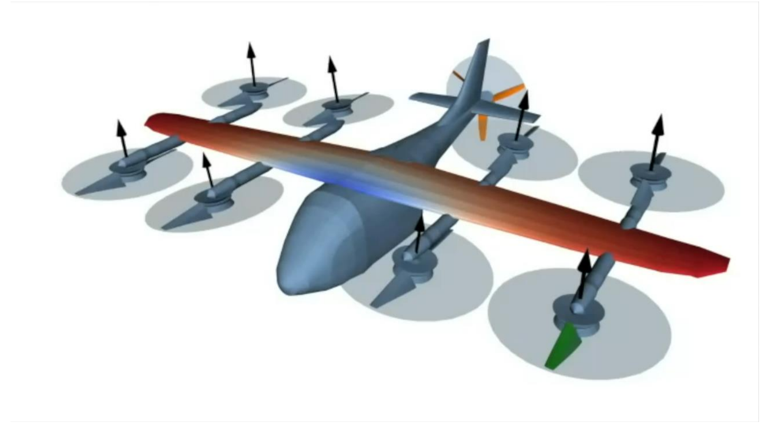
Equivalent circuit motor model





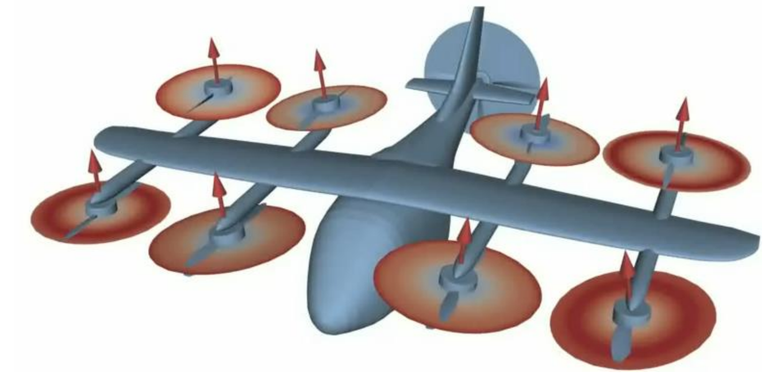
890
operations

Model used in the TC1 problem

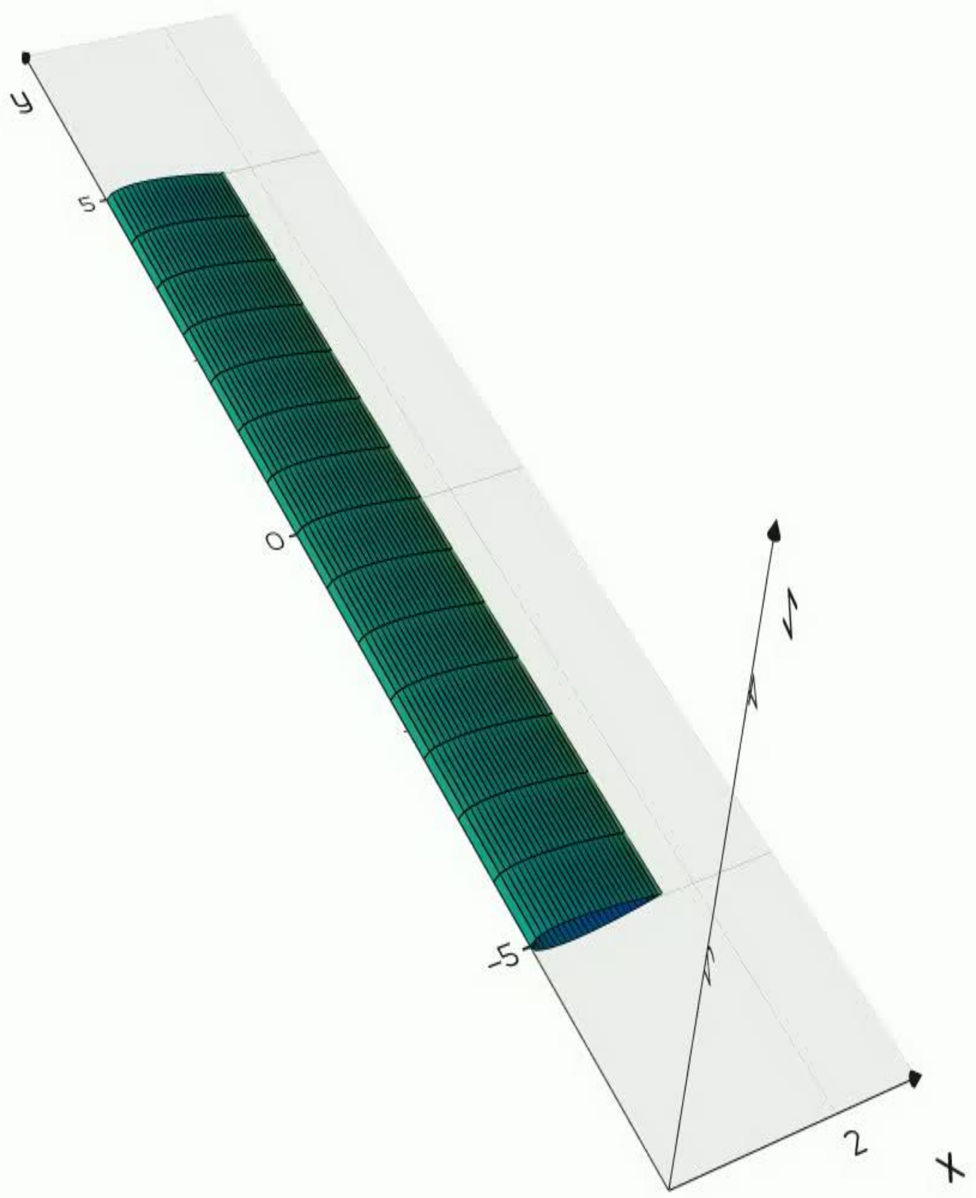


~20,000
operations

Model used in the TC2 problem



~200,000
operations


y
5
0
-5
2
x

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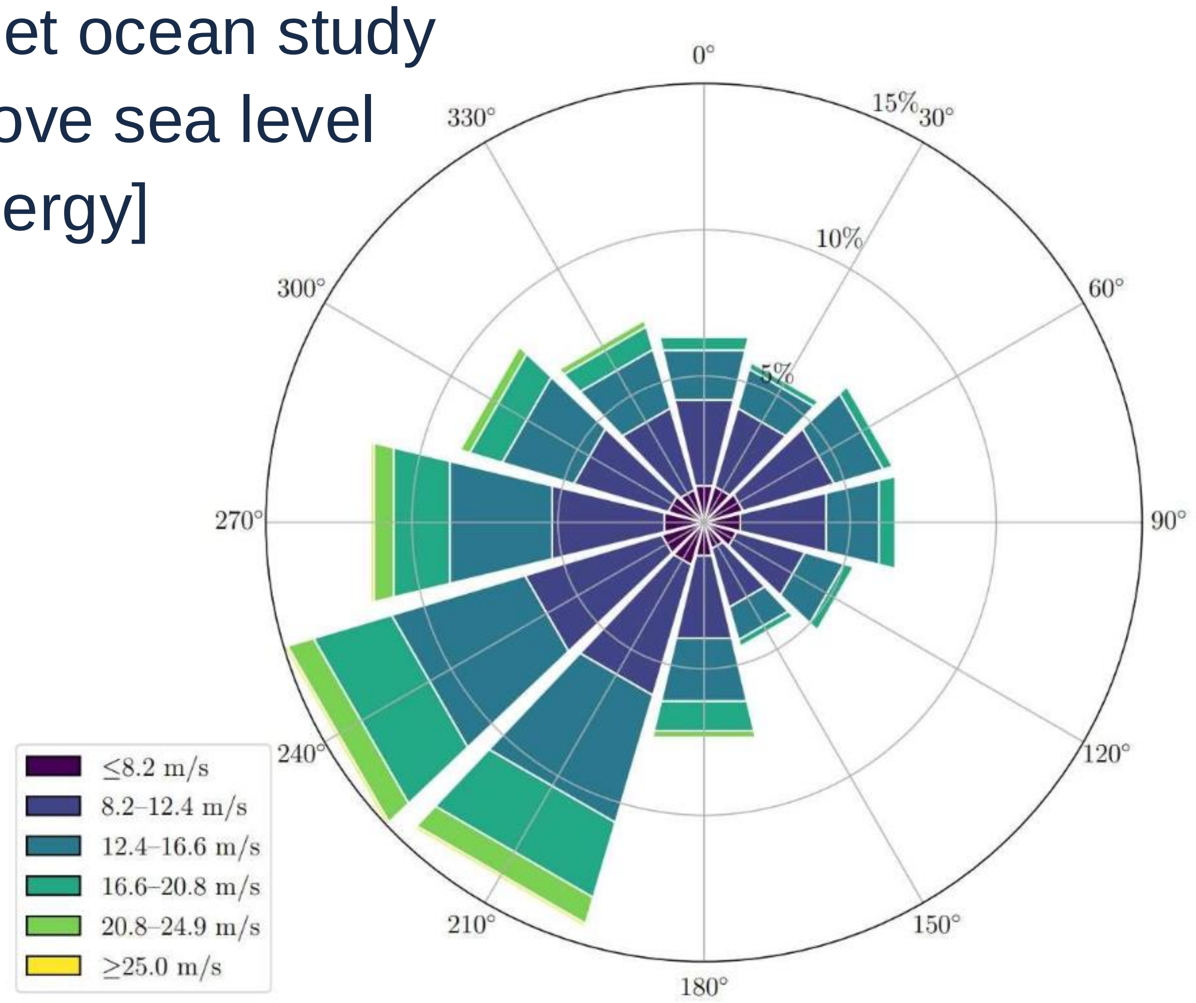
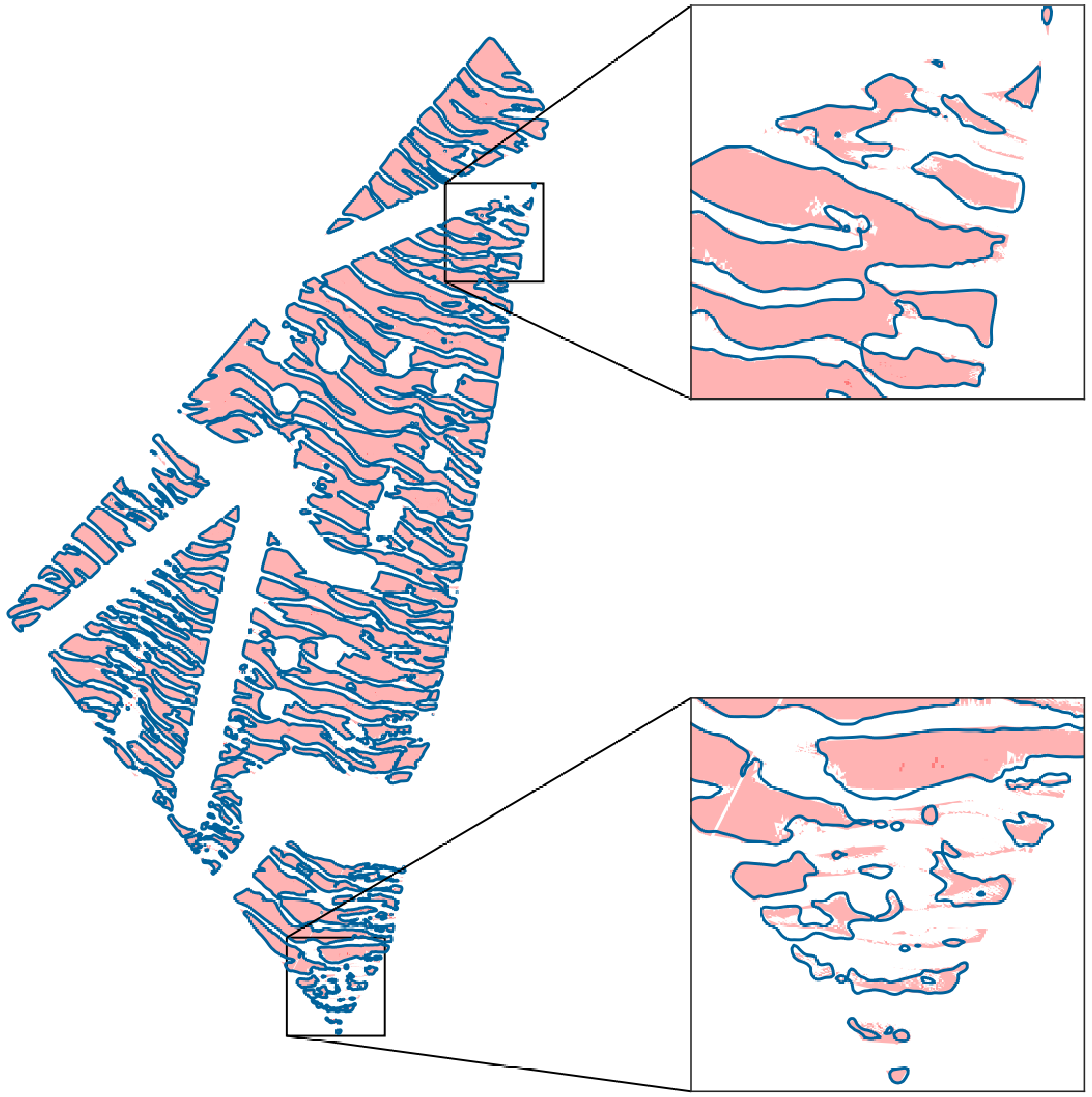
Towards large-scale MDO under uncertainty

Application: Offshore wind farm layout optimization using a novel shape constraint formulation

Data from met ocean study
at 100 m above sea level
[DNV GL Energy]

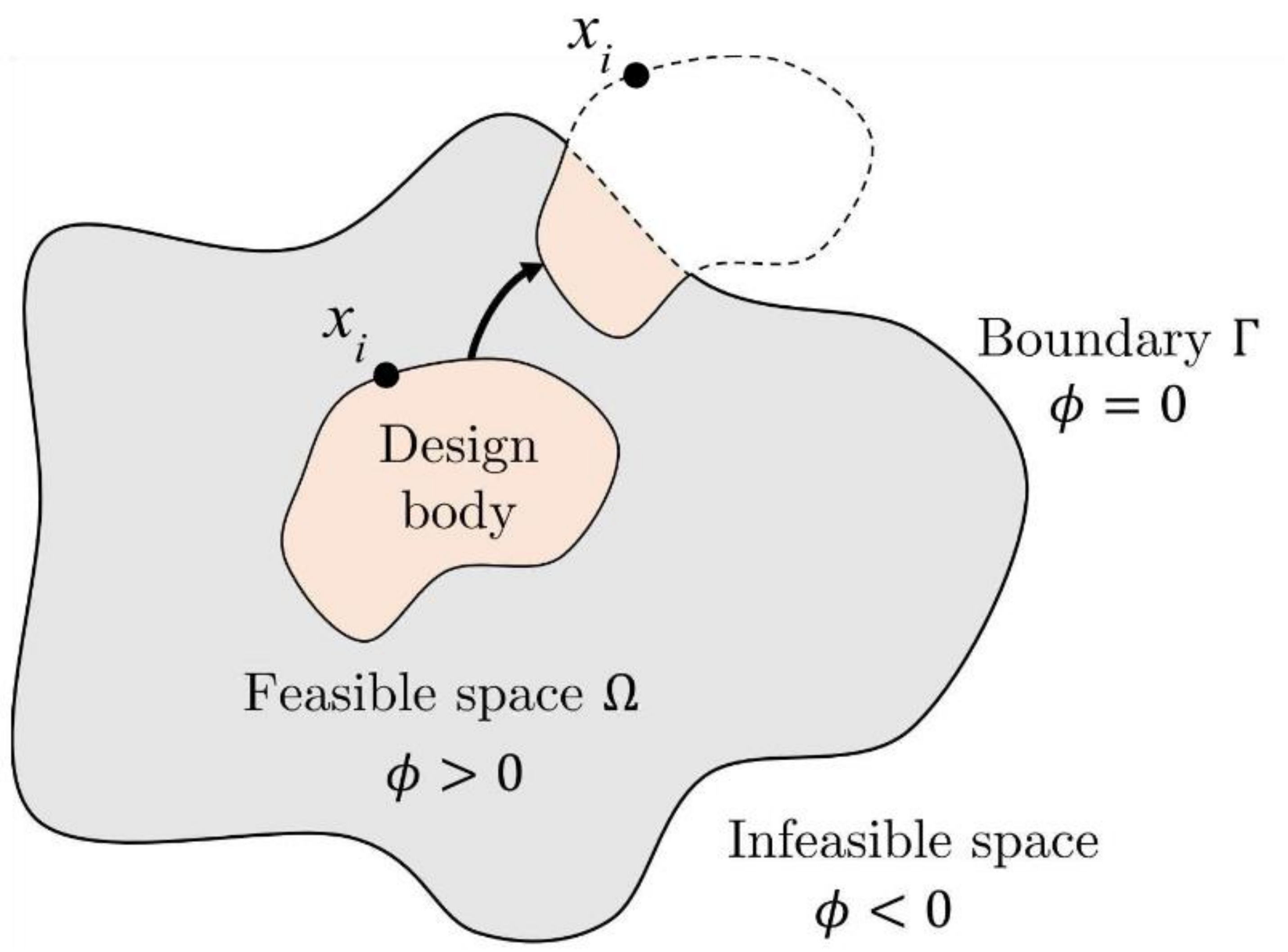
Hollandse Kust (West) Site

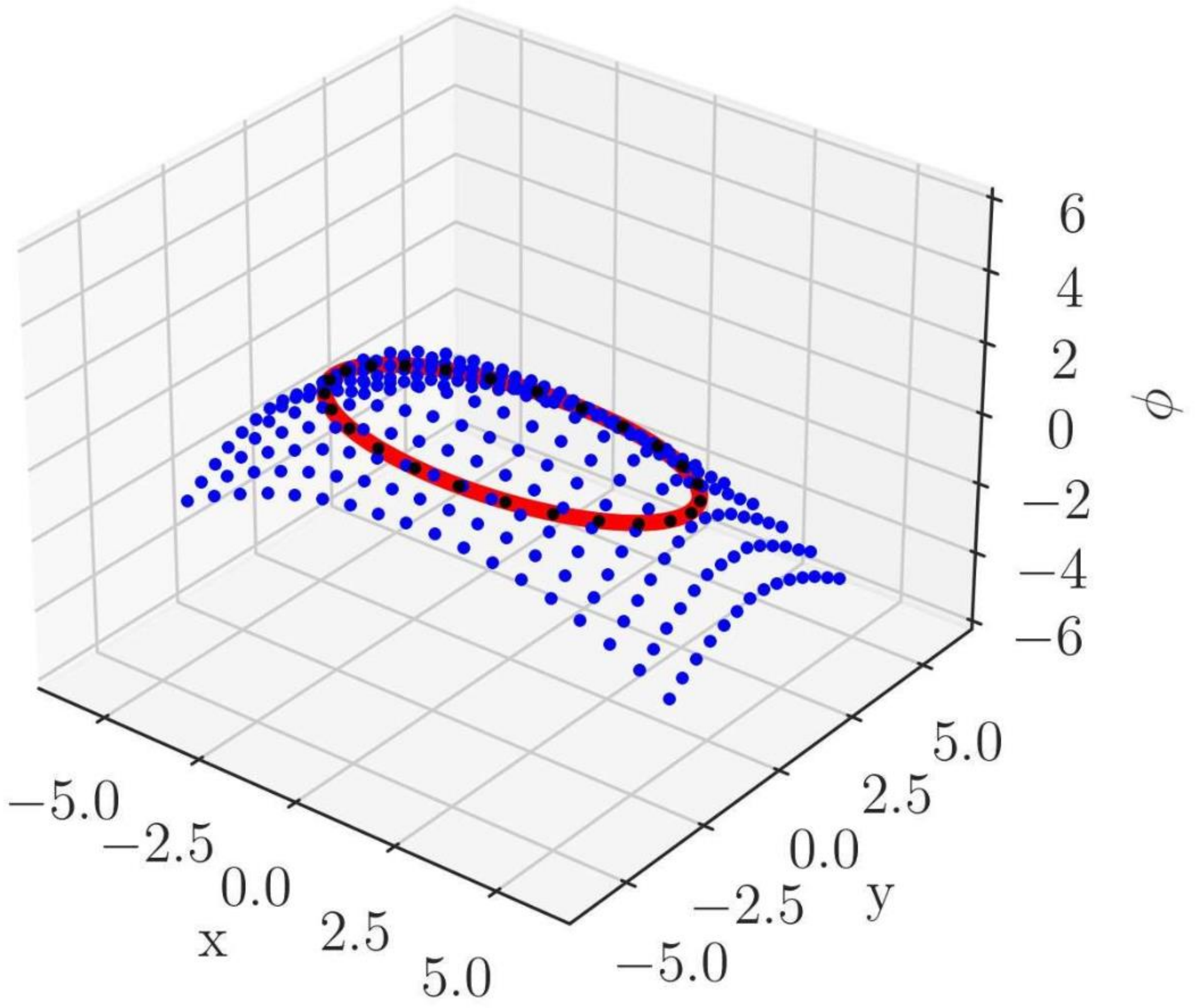


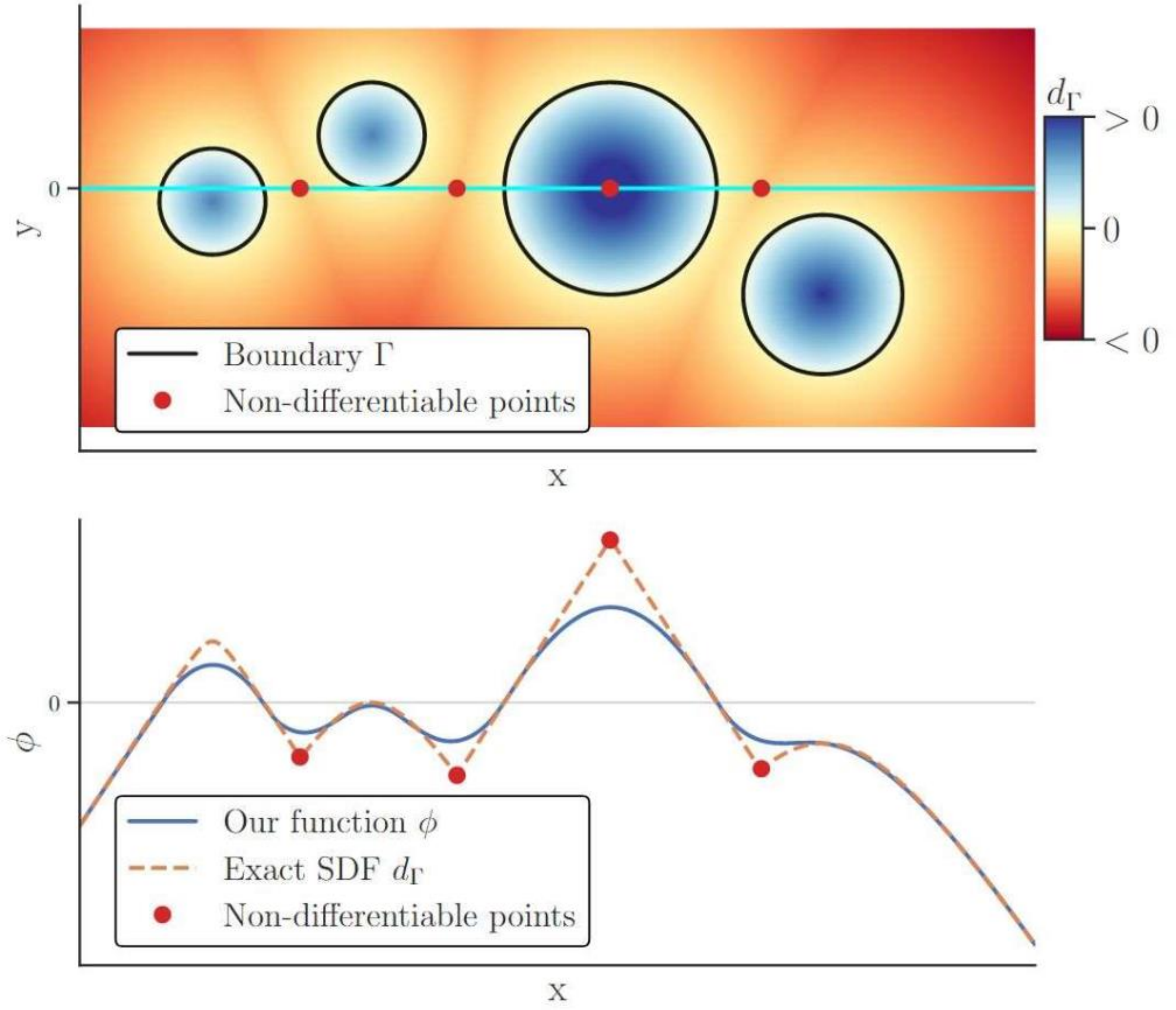


Graph-based modeling accelerated implementation of a new method for enforcing shape constraints

New constraint-enforcement method using **level-set functions**





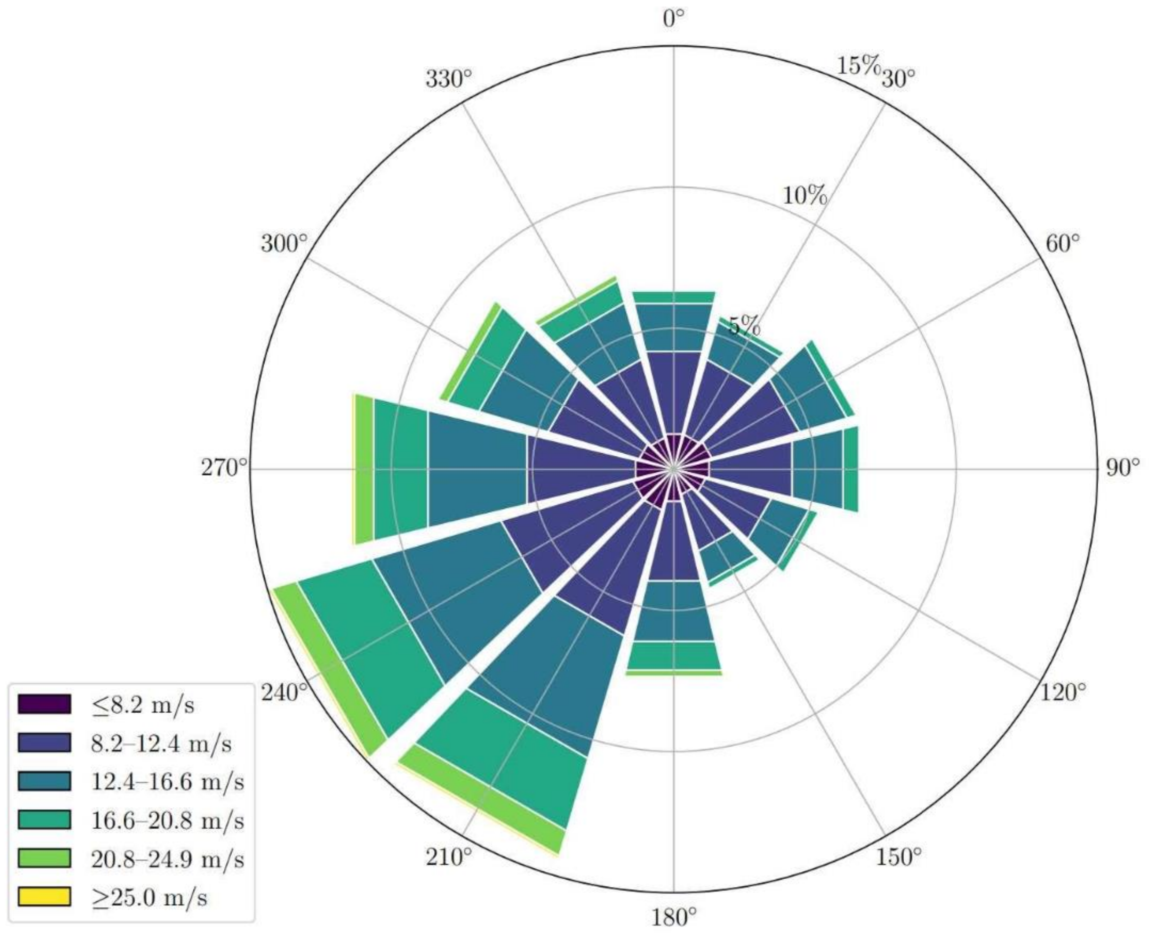


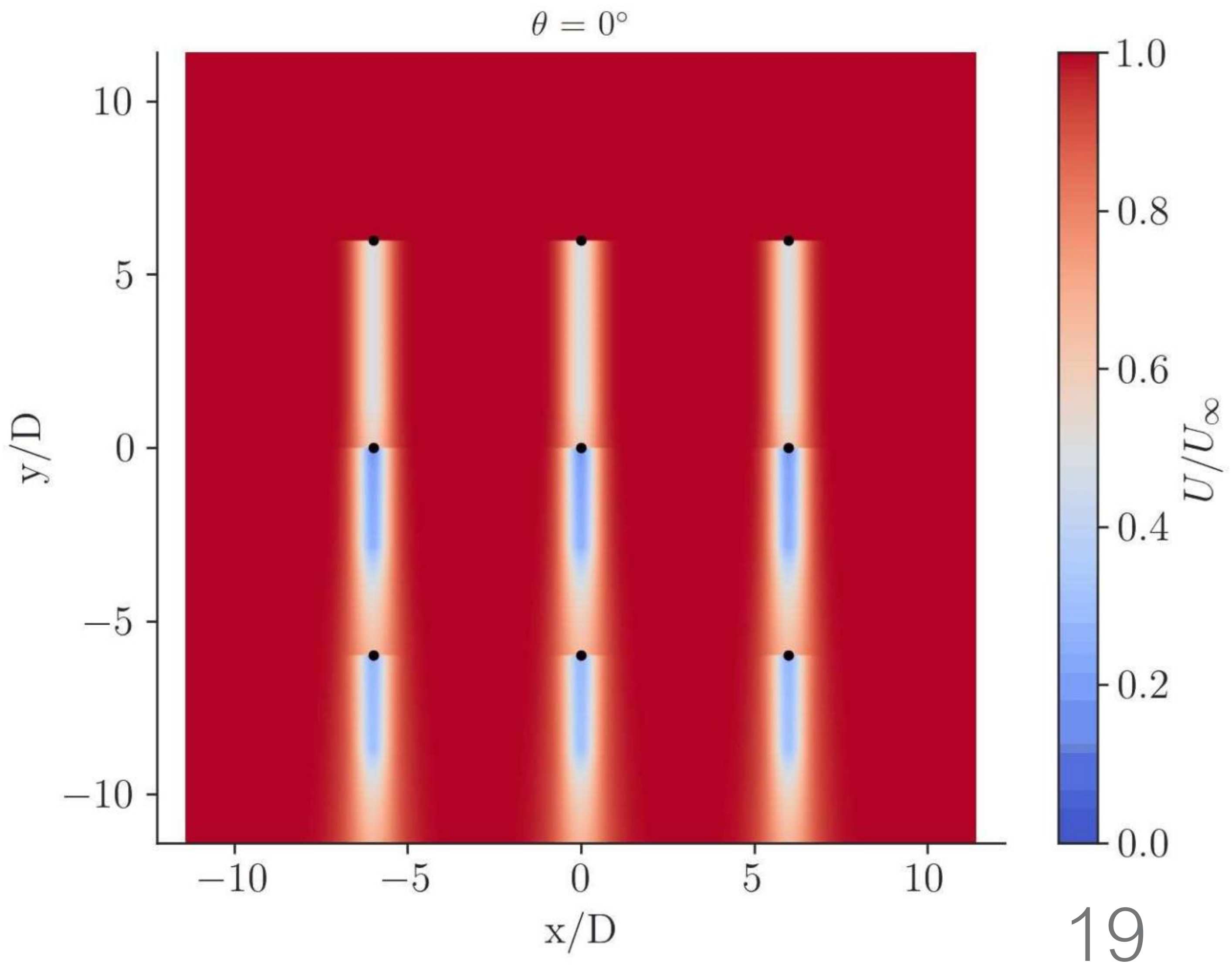
Design optimization problem

$$
\begin{aligned}
\text{maximize} \quad & \text{AEP} \\
\text{with respect to} \quad & \mathbf{x}, \mathbf{y} \in \mathbb{R}^{N_t} \\
\text{subject to} \quad & d_{ij} \geq d_{min} \\
& \phi(x_i, y_i) \geq 0
\end{aligned}
$$

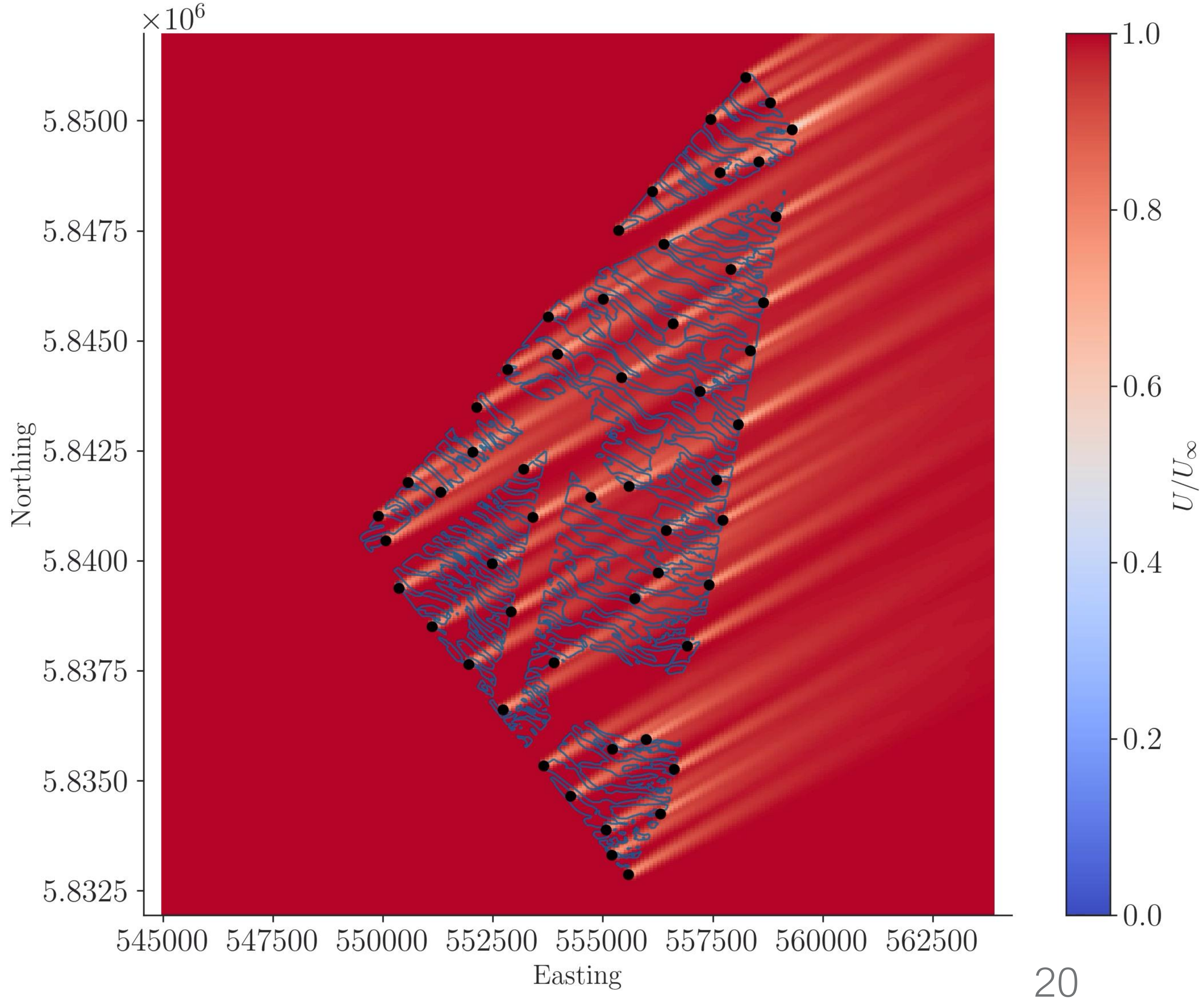
Spacing constraint:
Wind turbines maintain 1.8 rotor diameters distance space from each other

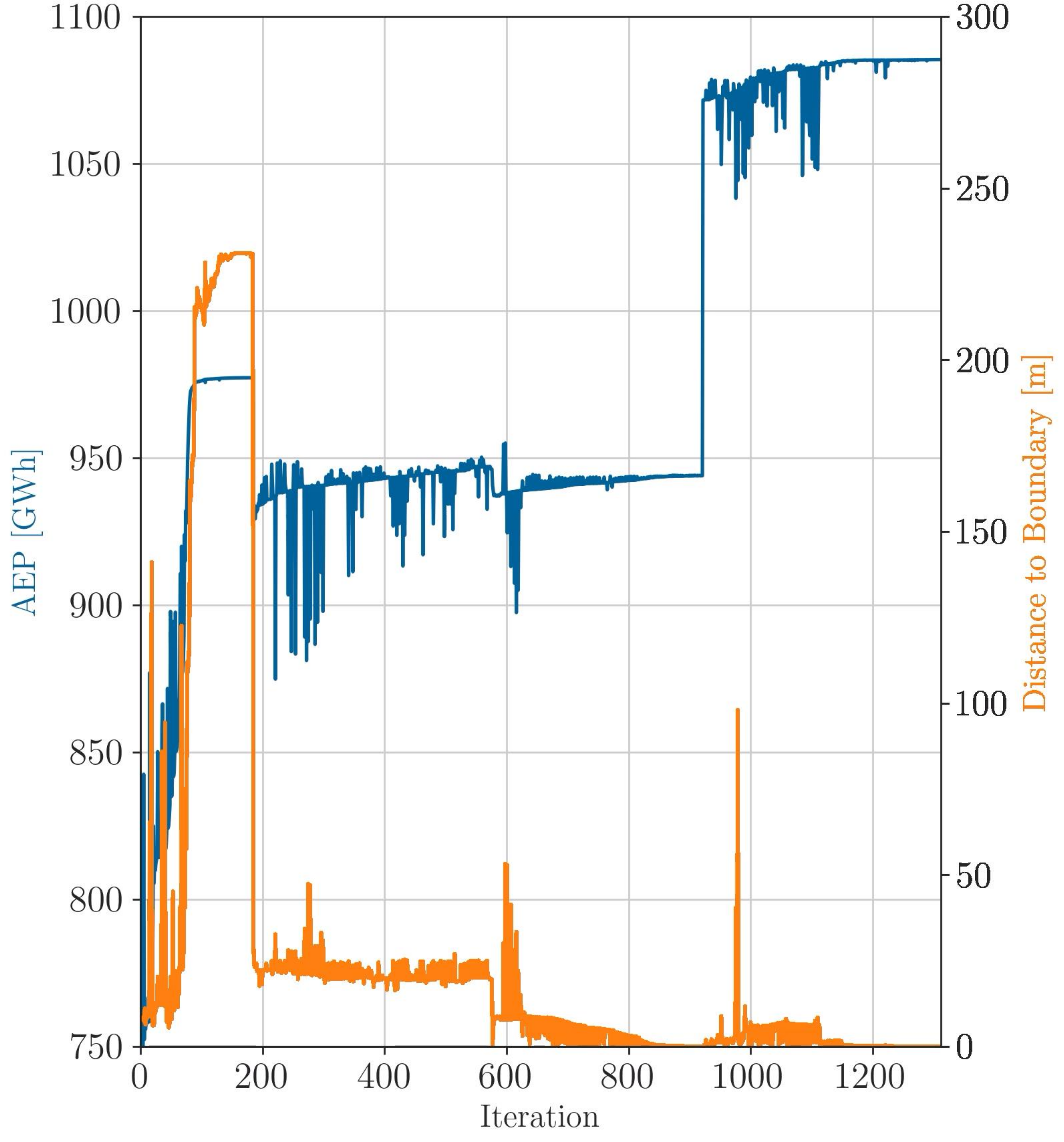
Boundary constraint:
Wind turbines must be within the feasible region



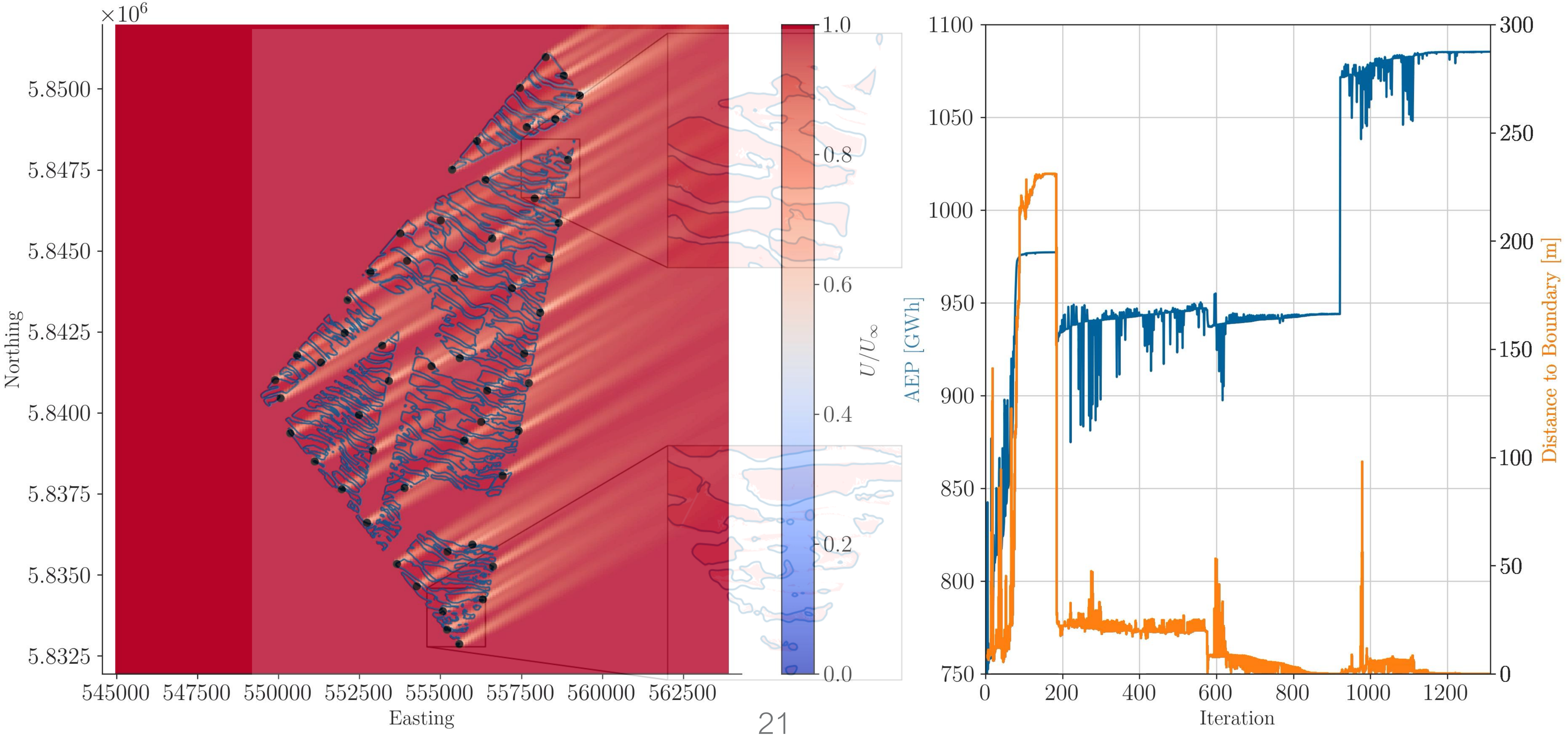


Optimization results





Optimization results



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Towards large-scale MDO under uncertainty

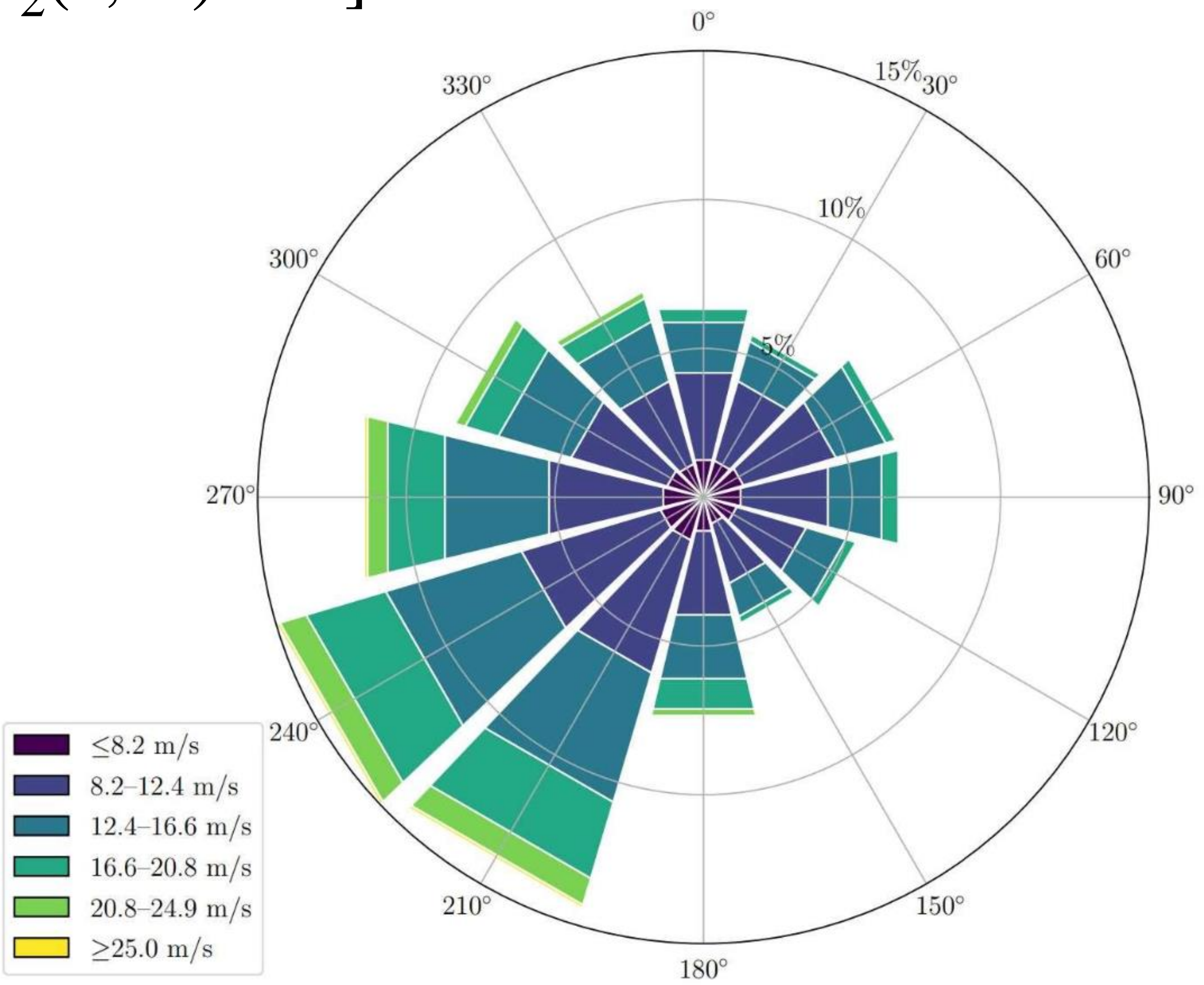
Large-scale MDO under uncertainty formally considers uncertain parameters in the problem formulations

Large-scale MDO

$$\min_x \quad \mathcal{F}(x)$$
$$\text{s.t.} \quad \mathcal{C}(x) < 0$$

Large-scale MDO under uncertainty

$$\min_x \quad \mathcal{M}(x) := \mathbb{E}[\mathcal{F}(x,U)] + \alpha\mathbb{S}[\mathcal{F}(x,U)]$$
$$\text{s.t.} \quad \mathcal{N}_1(x) := \mathbb{E}[\mathcal{C}_1(x,U)] + \alpha\mathbb{S}[\mathcal{C}_1(x,U)] < 0$$
$$\mathcal{N}_2(x) := \mathbb{P}[\mathcal{C}_2(x,U) < 0] > P$$



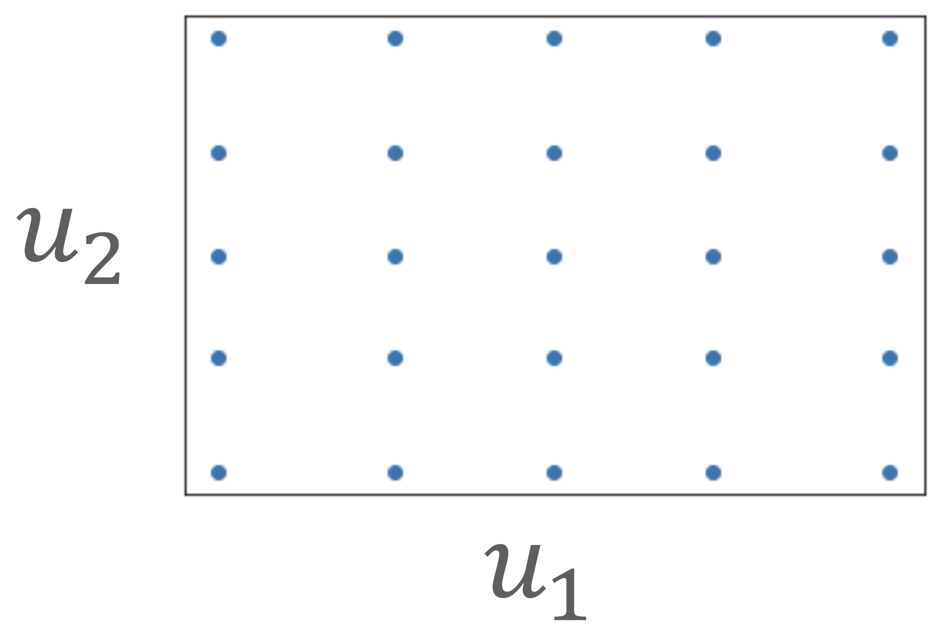
Accelerated Model evaluations on Tensor grids using Computational graph transformations (AMTC)

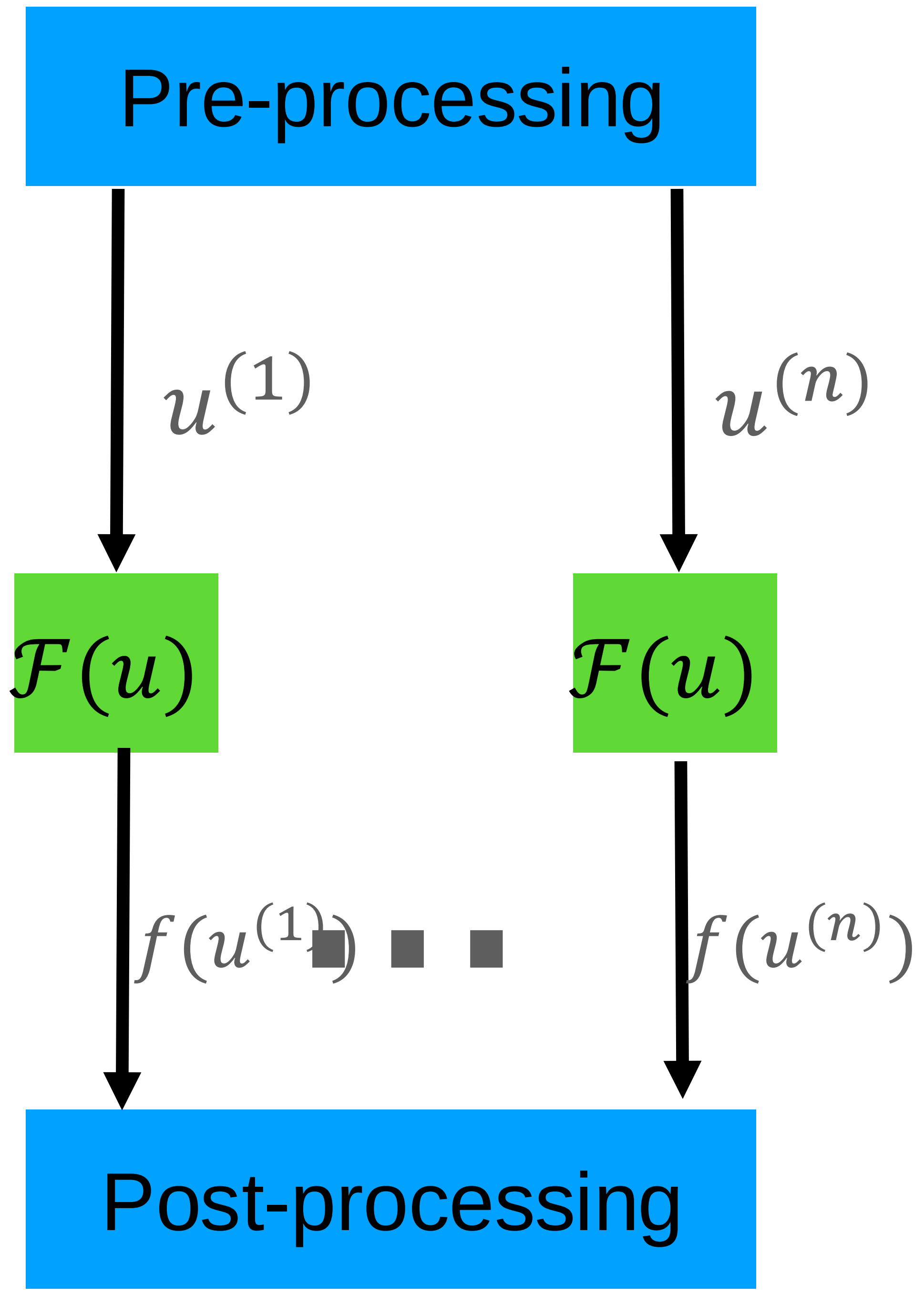
Example:

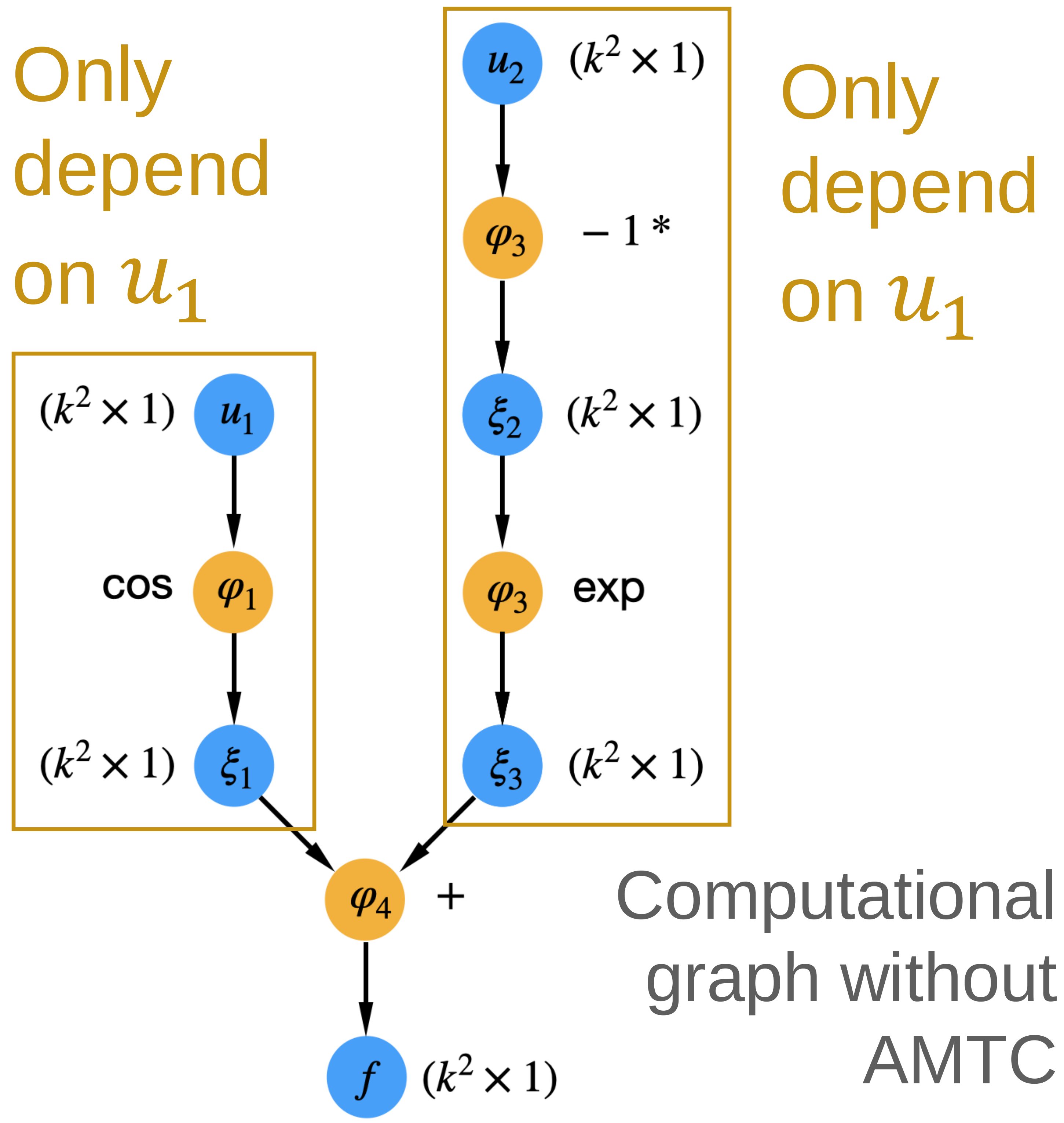
- Function:

$$f = cos(u_1) + exp(-u_2)$$

- Inputs: $\boldsymbol{u} = \boldsymbol{u}_1^k \times \boldsymbol{u}_2^k$



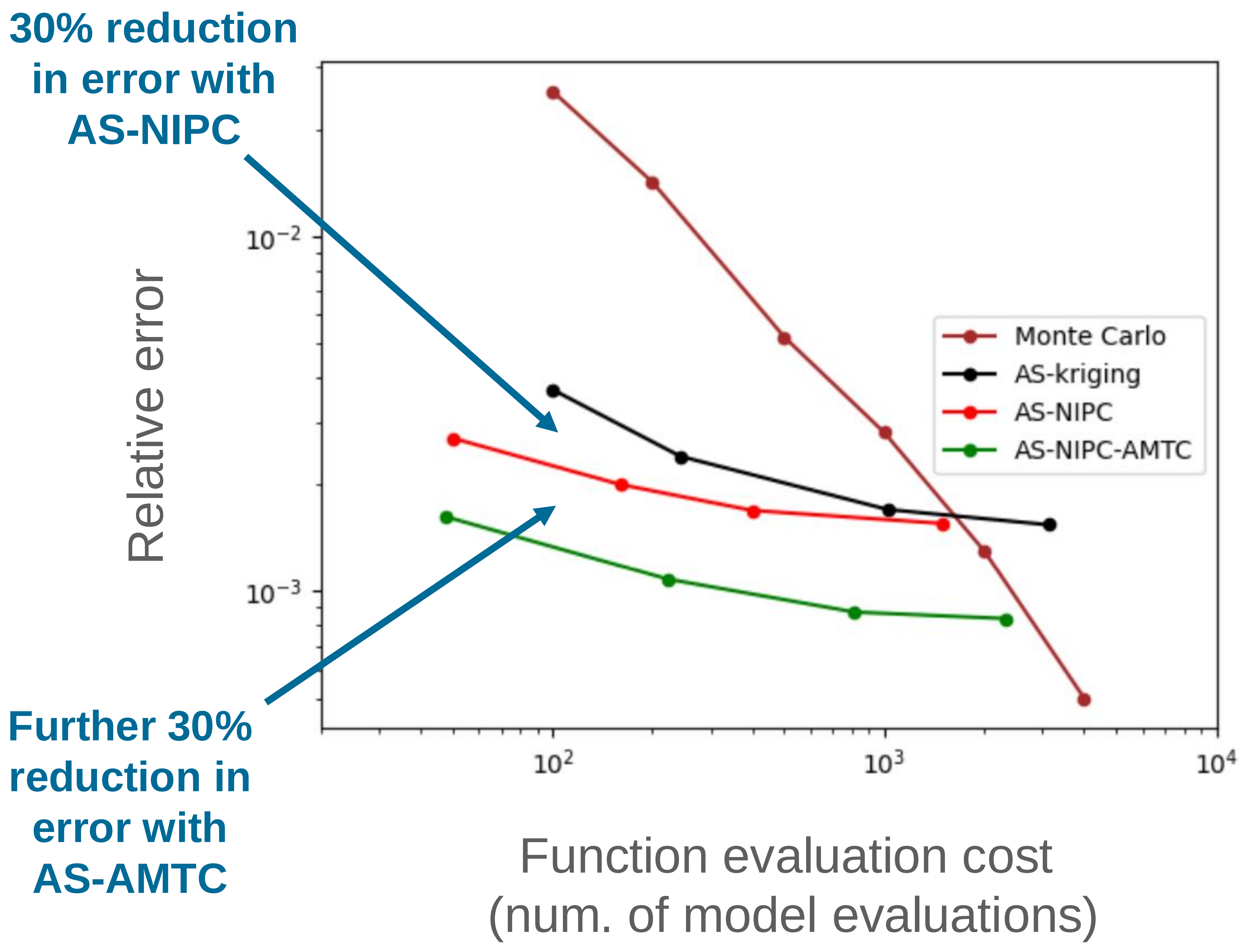


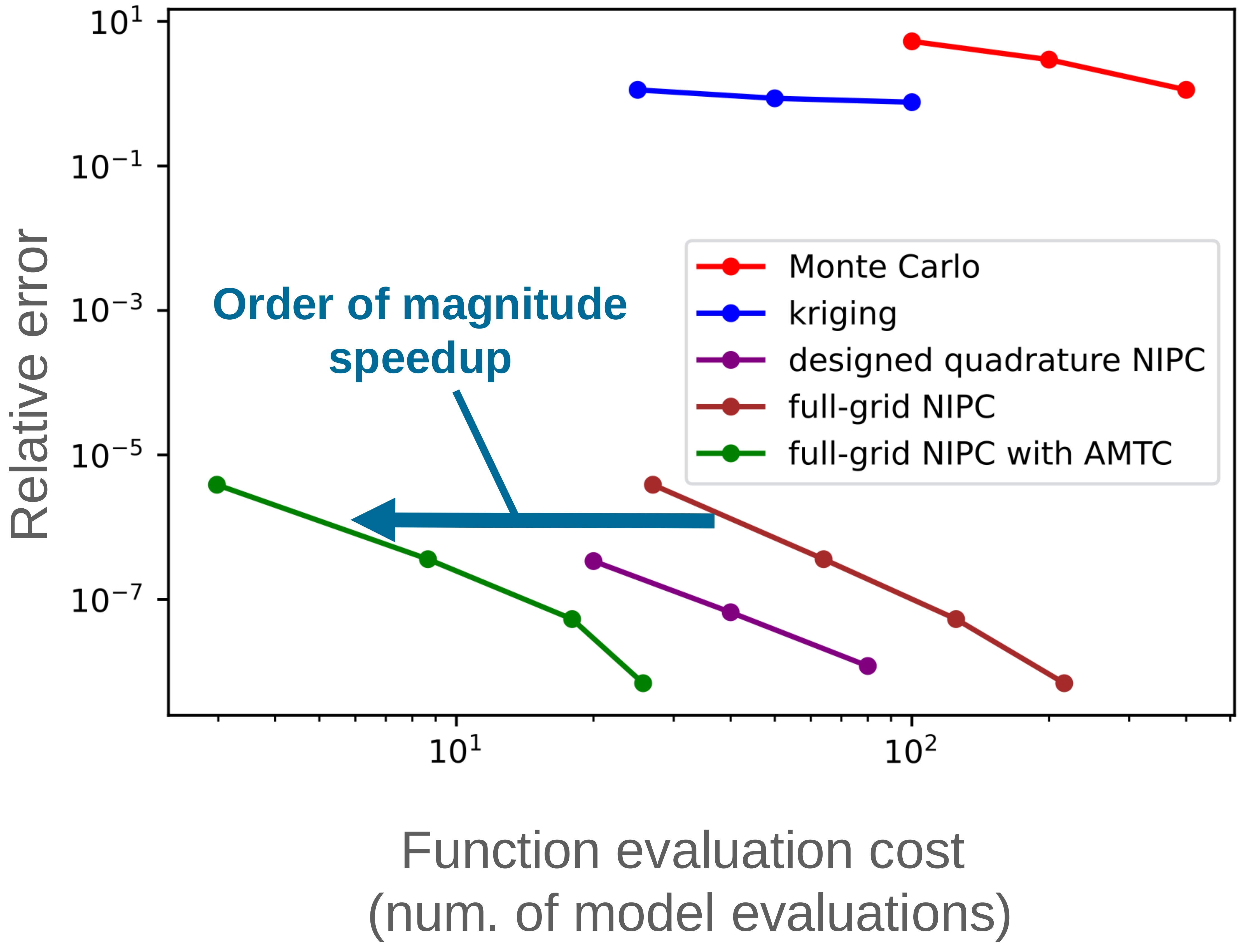


Computational graph without AMTC

Wang, B., Sperry, M., Gandarillas, V. E., & Hwang, J. T. (2024). Accelerating model evaluations in uncertainty propagation on tensor grids using computational graph transformations. Aerospace Science and Technology, 145, 108843.

AMTC provides significant speedups





Summary and takeaways

Large-scale MDO techniques have matured in the past decade

Gradient computation has been the biggest challenge

Graph-based modeling reduces the adoption barrier

Large-scale MDO under uncertainty is becoming feasible

Opportunities to apply these methods to **wind energy system design**

Thank you!

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