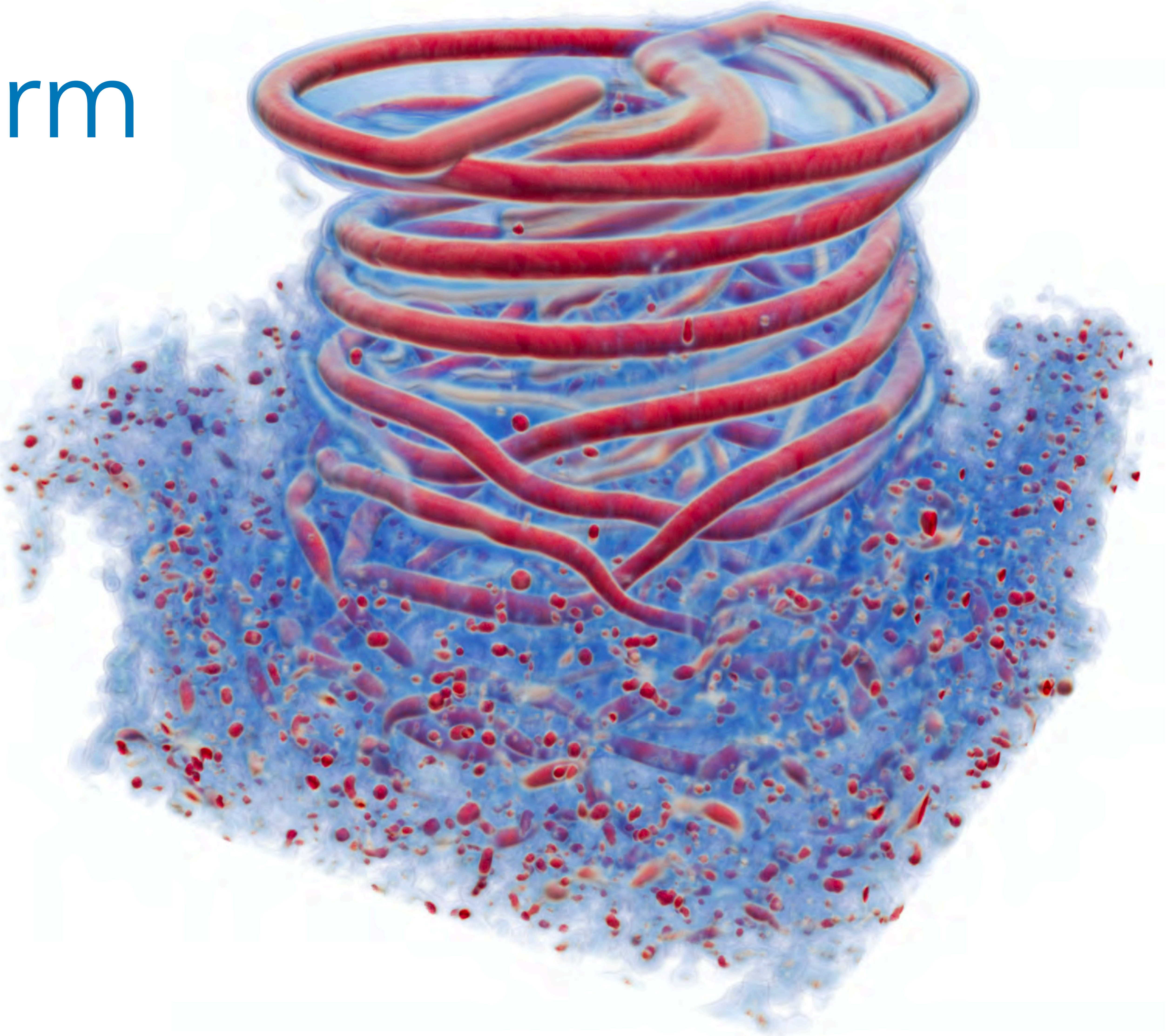
Derivatives for Wind Turbine and Wind Farm Design Optimization



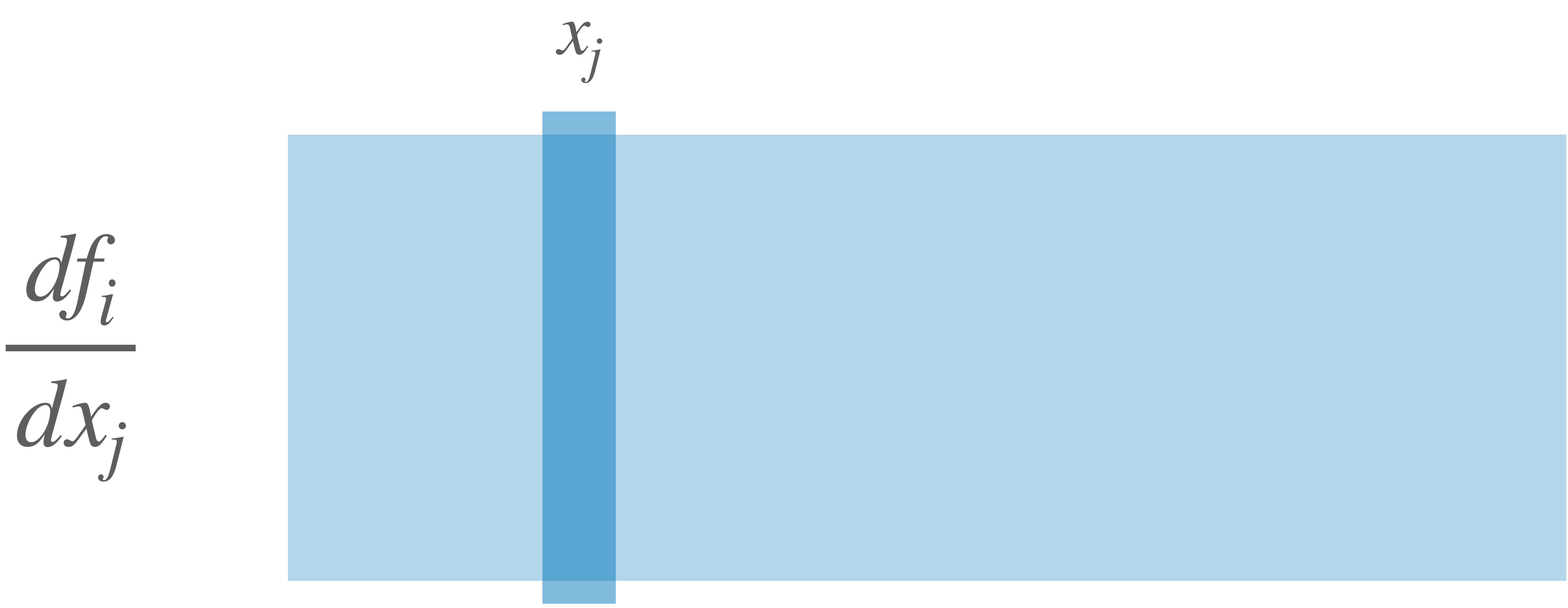
Wind Energy Sys. Eng. Workshop
August 2022

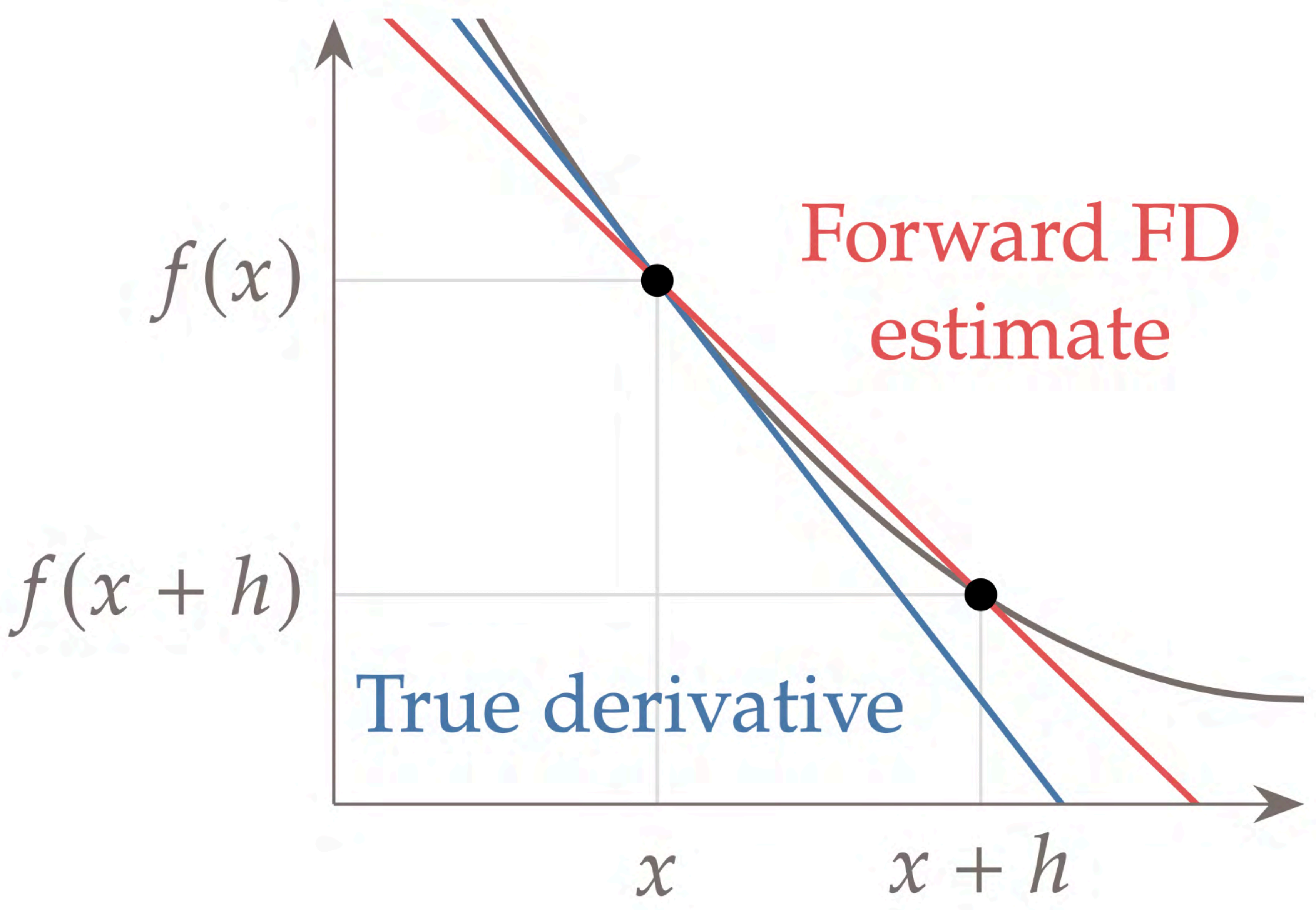
Brigham Young University
Andrew Ning

Derivatives

$$\frac{df_i}{dx_j}$$

Derivatives

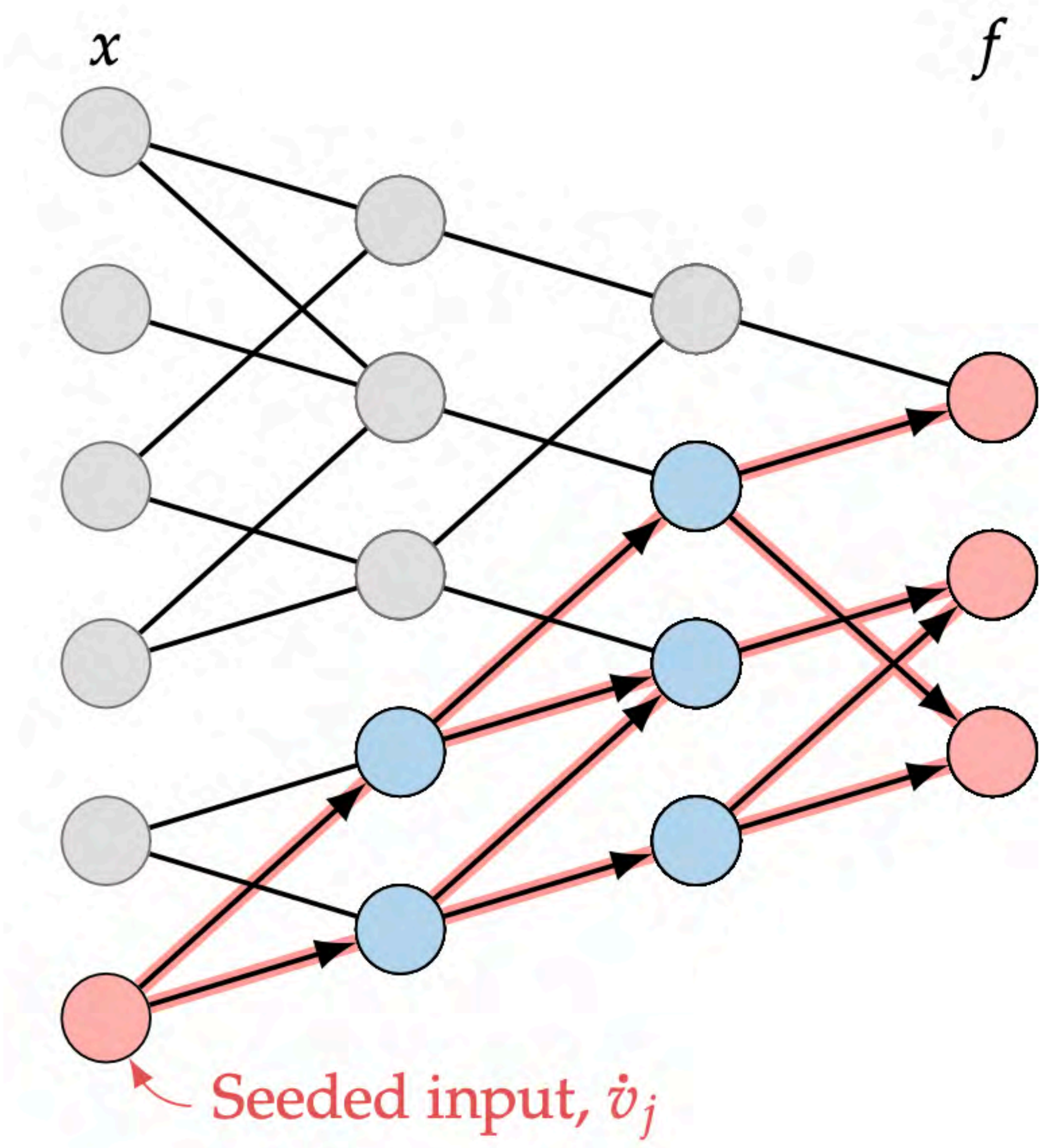




Finite differencing

Derivatives

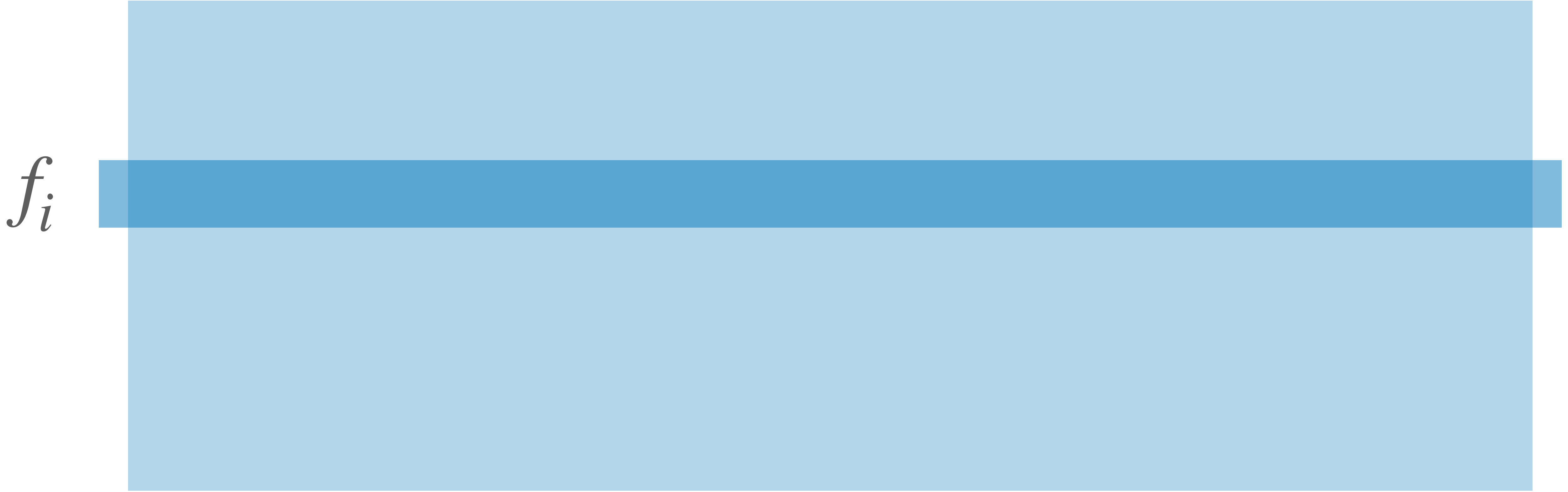
$$\frac{df_i}{dx_j}$$

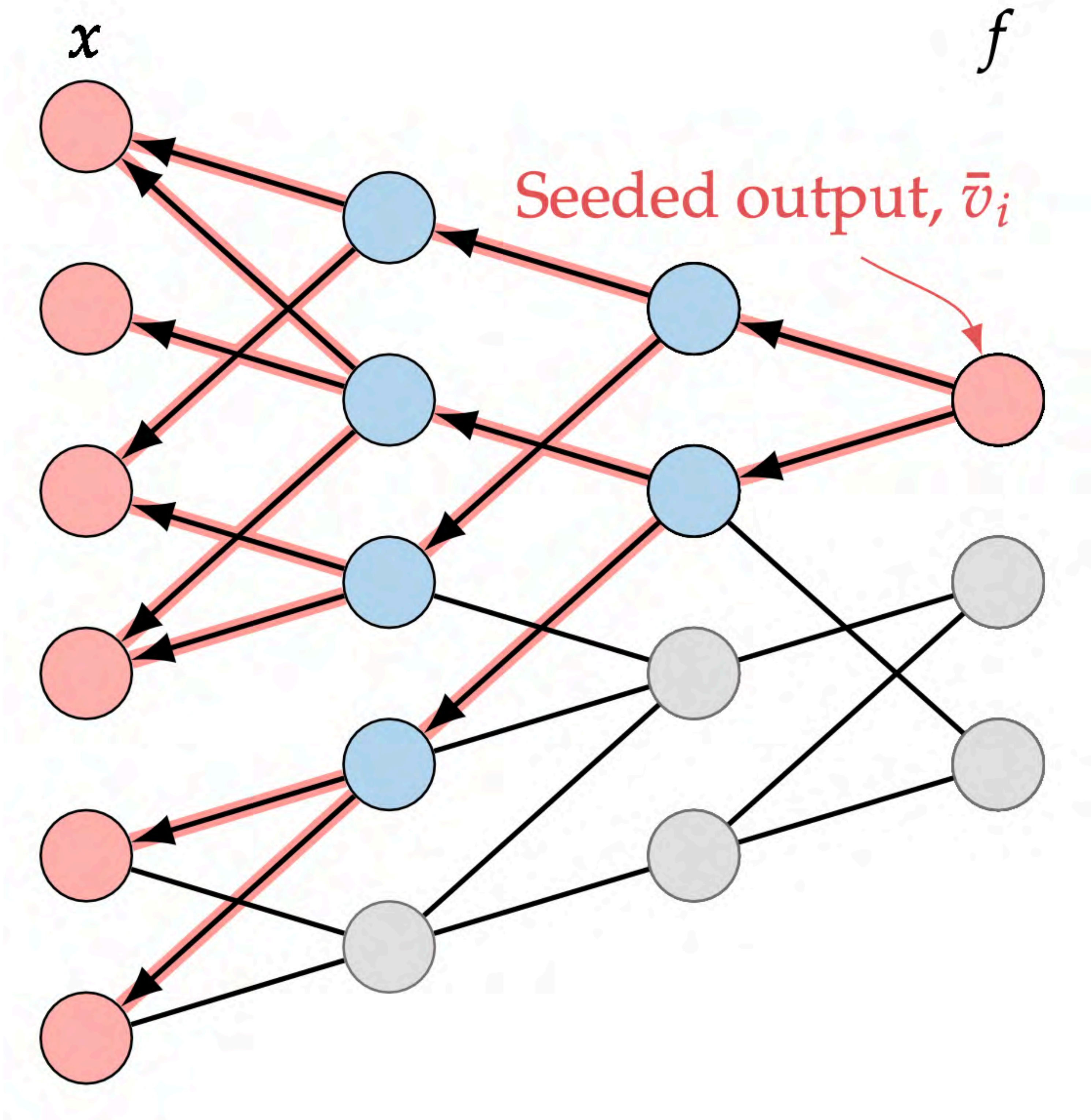


Forward Mode
Algorithmic Differentiation

Derivatives

$$\frac{df_i}{dx_j}$$

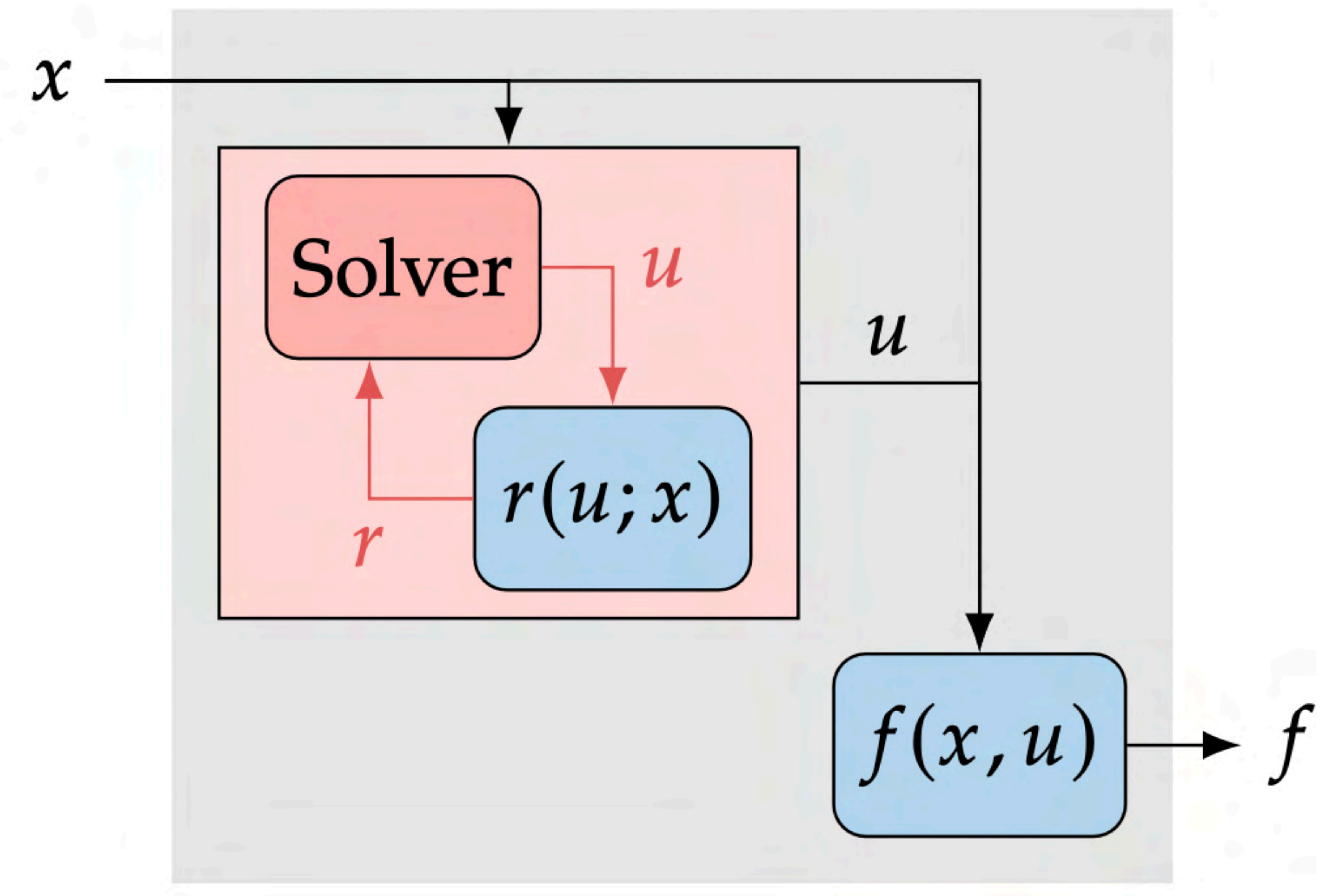




Reverse Mode
Algorithmic Differentiation

Derivatives

$$\frac{df_i}{dx_j}$$



Direct and Adjoint Methods

A Simple Wind Turbine Example

maximize	annual energy production
by varying	chord_i for $i = 1 \ldots 5$
	twist_i for $i = 1 \ldots 4$
	tip-speed ratio
	pitch_i for $i = 1 \ldots 80$
subject to	$\text{power}_i \leq 5$ MW for $i = 1 \ldots 80$
	$\text{thrust}_i \leq 600$ kN for $i = 1 \ldots 80$
	flapwise $\text{loads}_j \leq 6500$ N/m for $j = 1 \ldots 17$
	$\text{pitch}_{i+1} > \text{pitch}_i$ for $i = 1 \ldots 79$

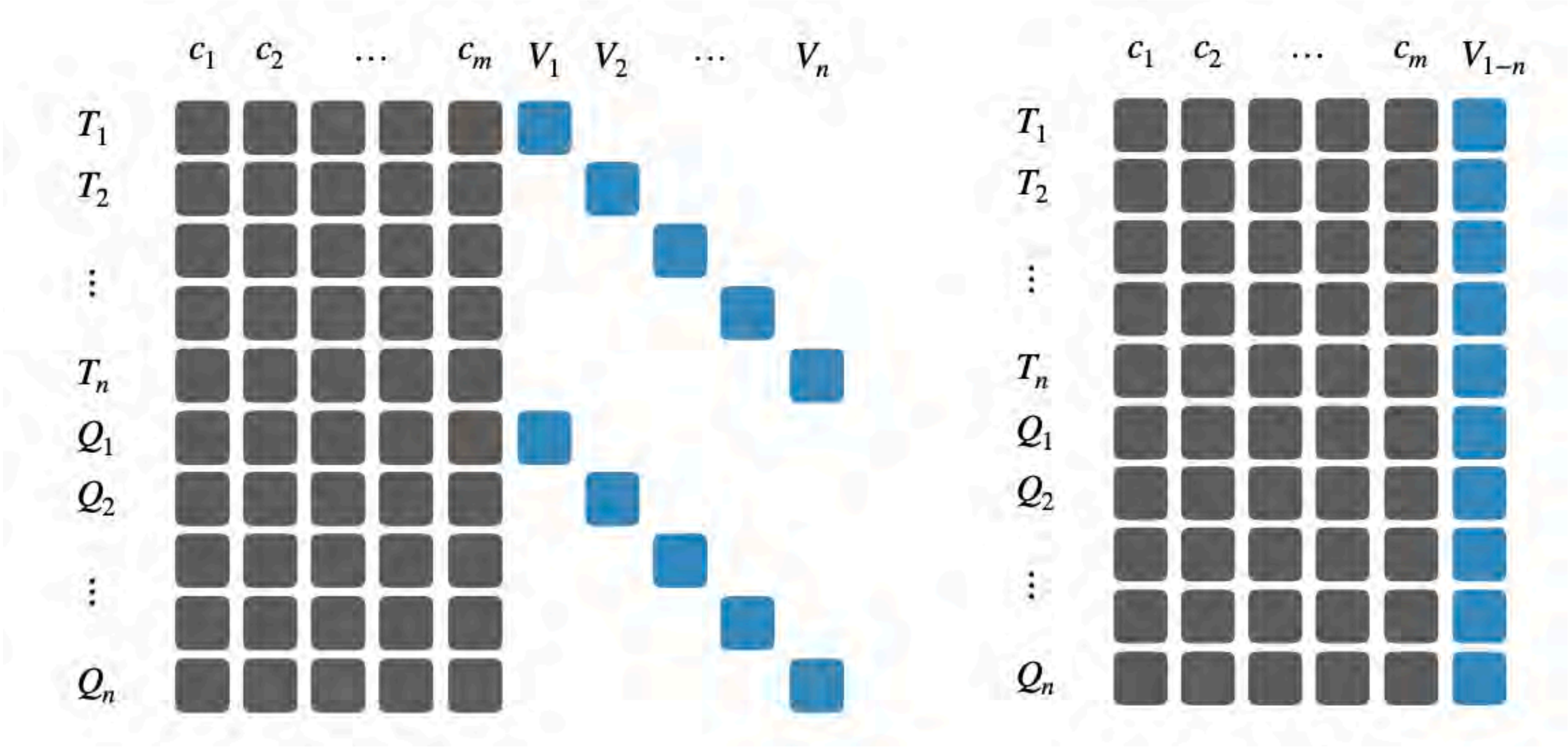
Method	Time (s)
Finite Differencing (FD)	112

Method	Time (s)
Finite Differencing (FD)	112
One Residual (FD)	67

Method	Time (s)
Finite Differencing (FD)	112
One Residual (FD)	67
One Residual with AD	24

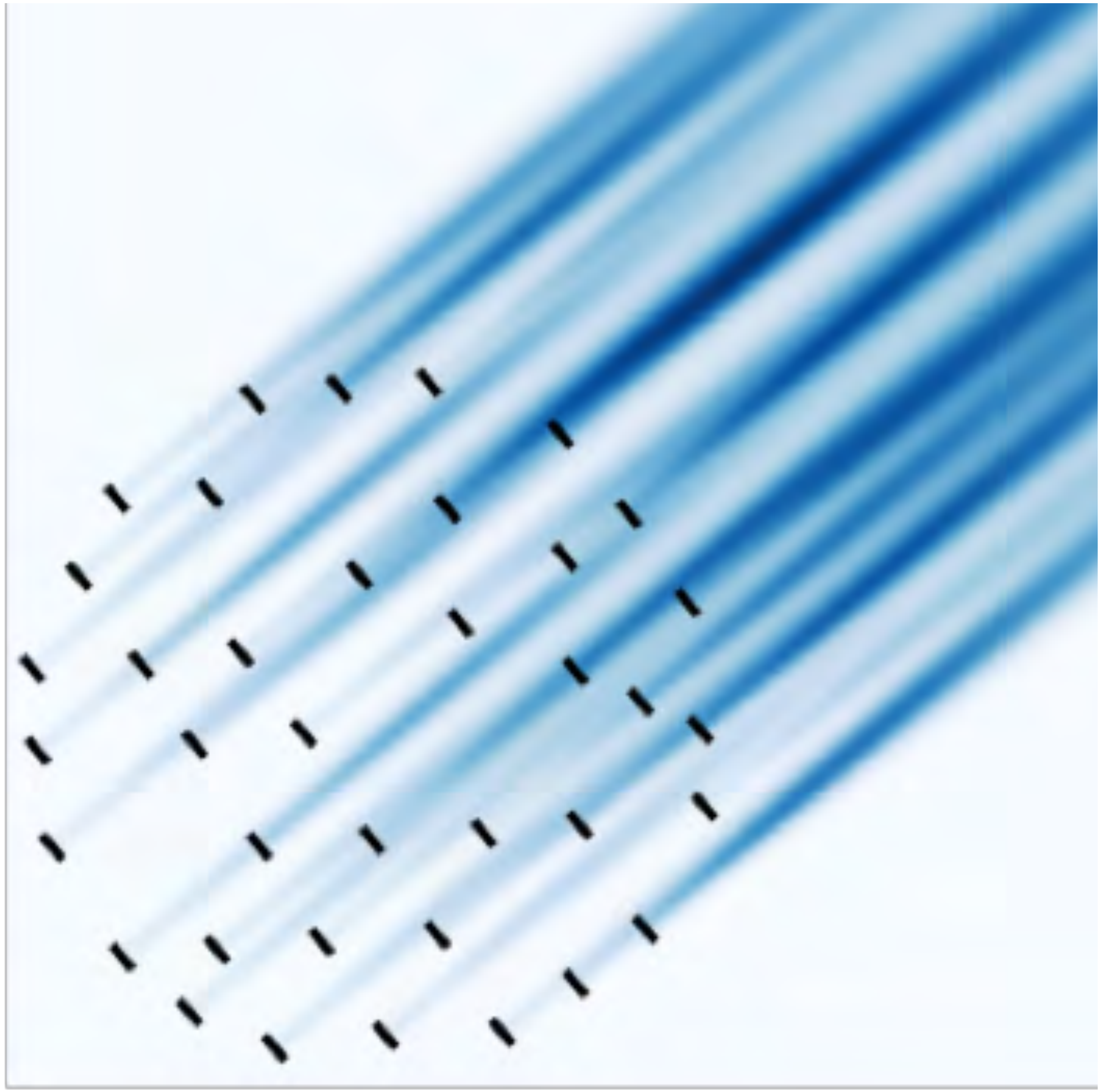
Method	Time (s)
Finite Differencing (FD)	112
One Residual (FD)	67
One Residual with AD	24
One Residual with AD/analytic	12

Method	Time (s)
Finite Differencing (FD)	112
One Residual (FD)	67
One Residual with AD	24
One Residual with AD/analytic	12
Sparse Jacobian	5

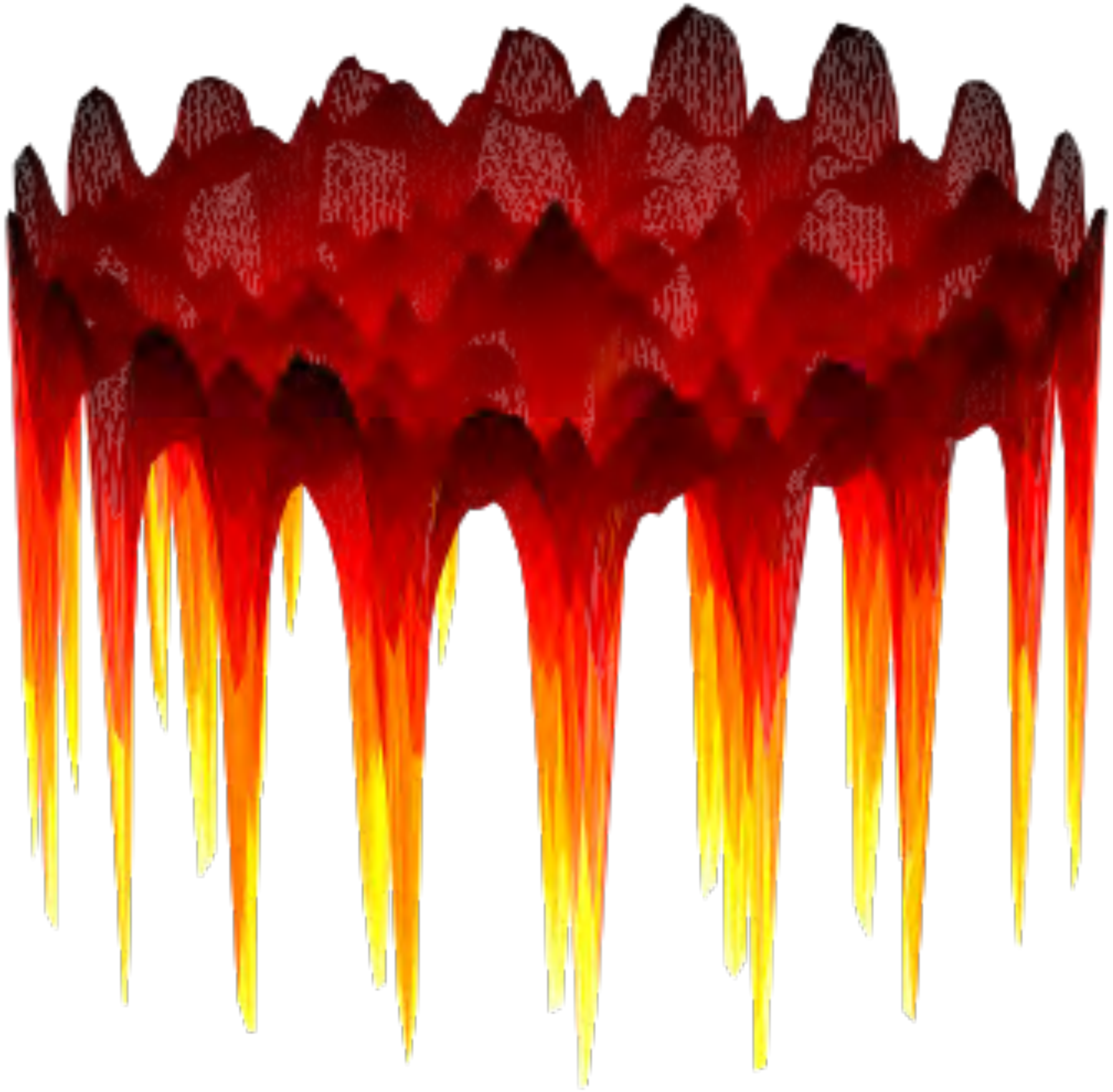


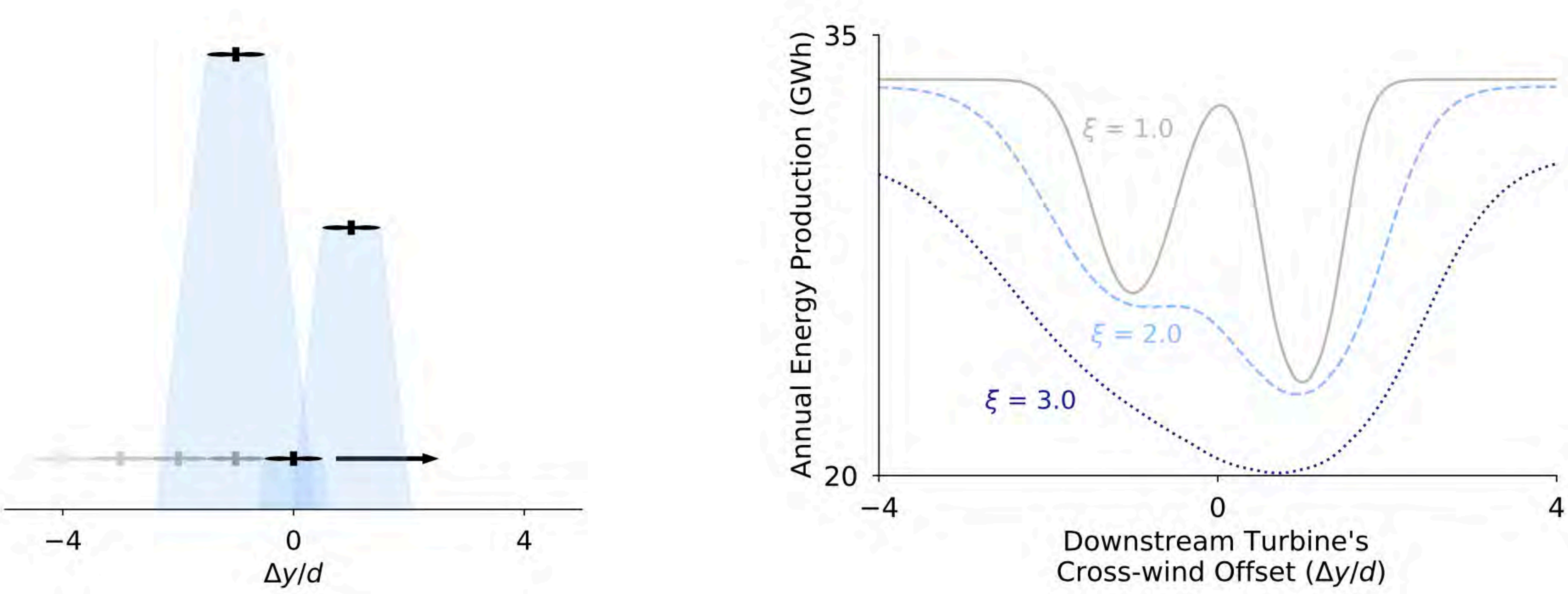
Ning, A., "Using Blade Element Momentum Methods with Gradient-Based Design Optimization," Structural and Multidisciplinary Optimization, Vol. 64, No. 2, May 2021, pp. 994–1014.

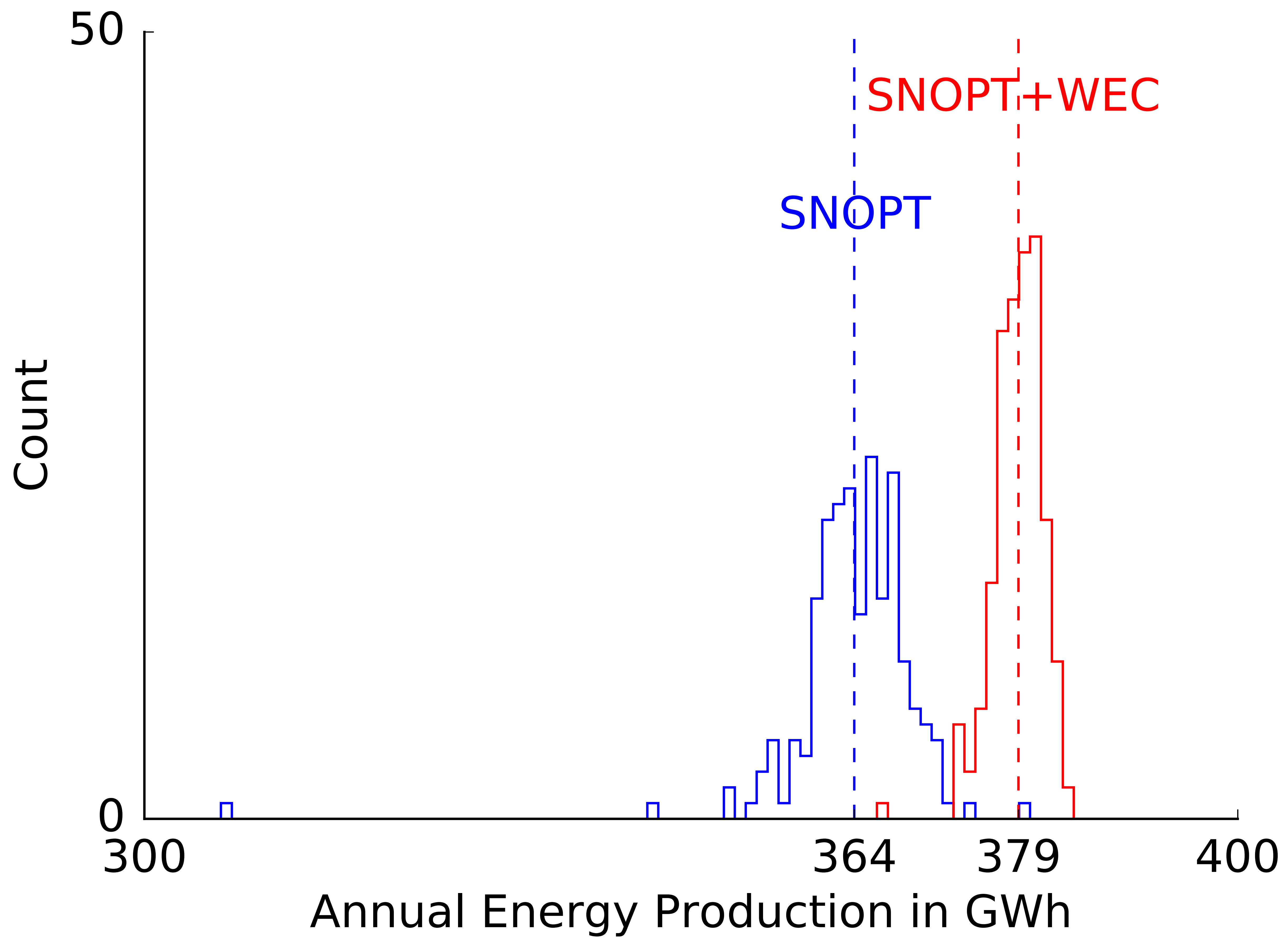
Wind Farm Optimization



Searching Multimodal Design Spaces in Wind Farm Layout Optimization

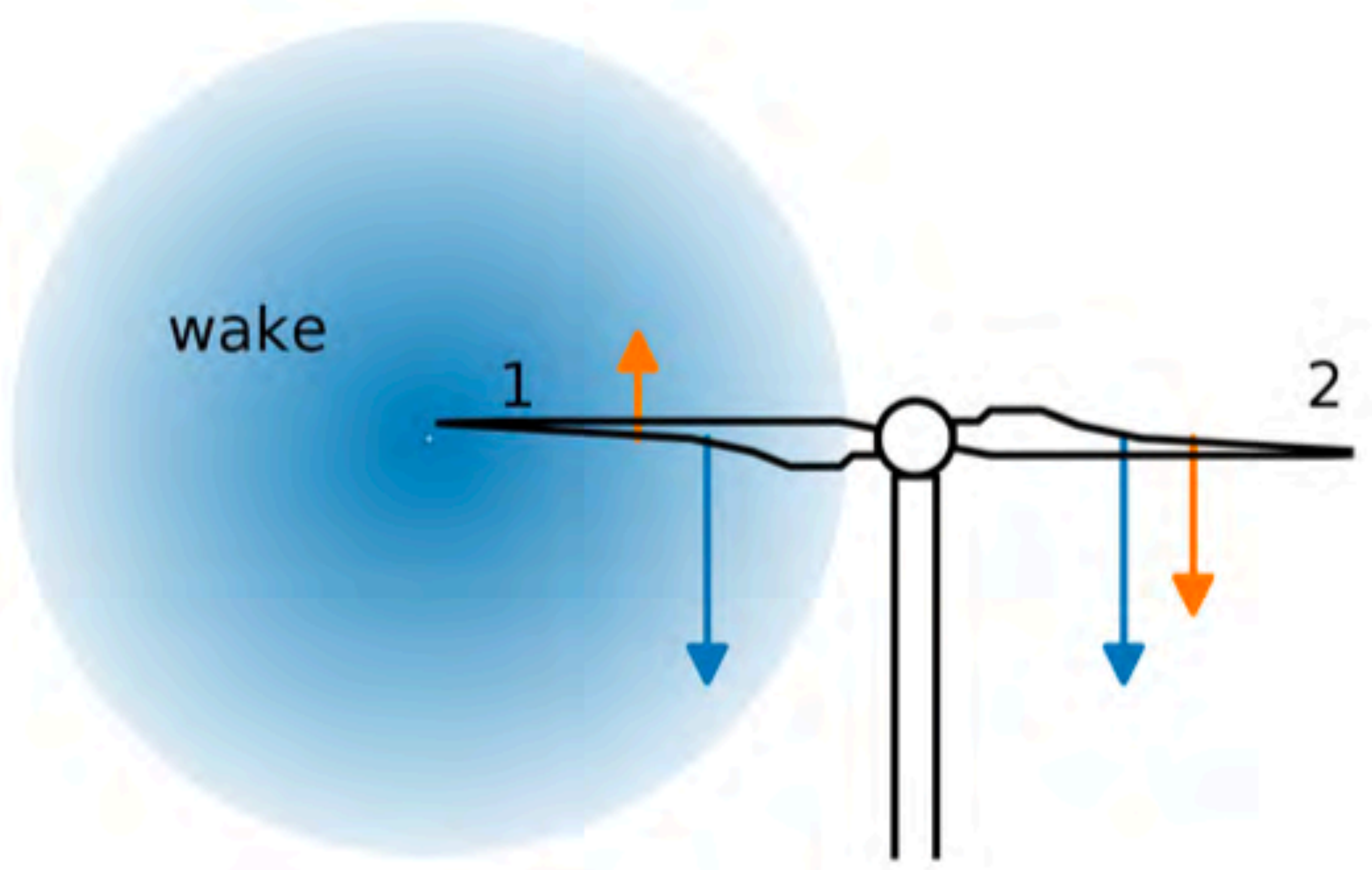


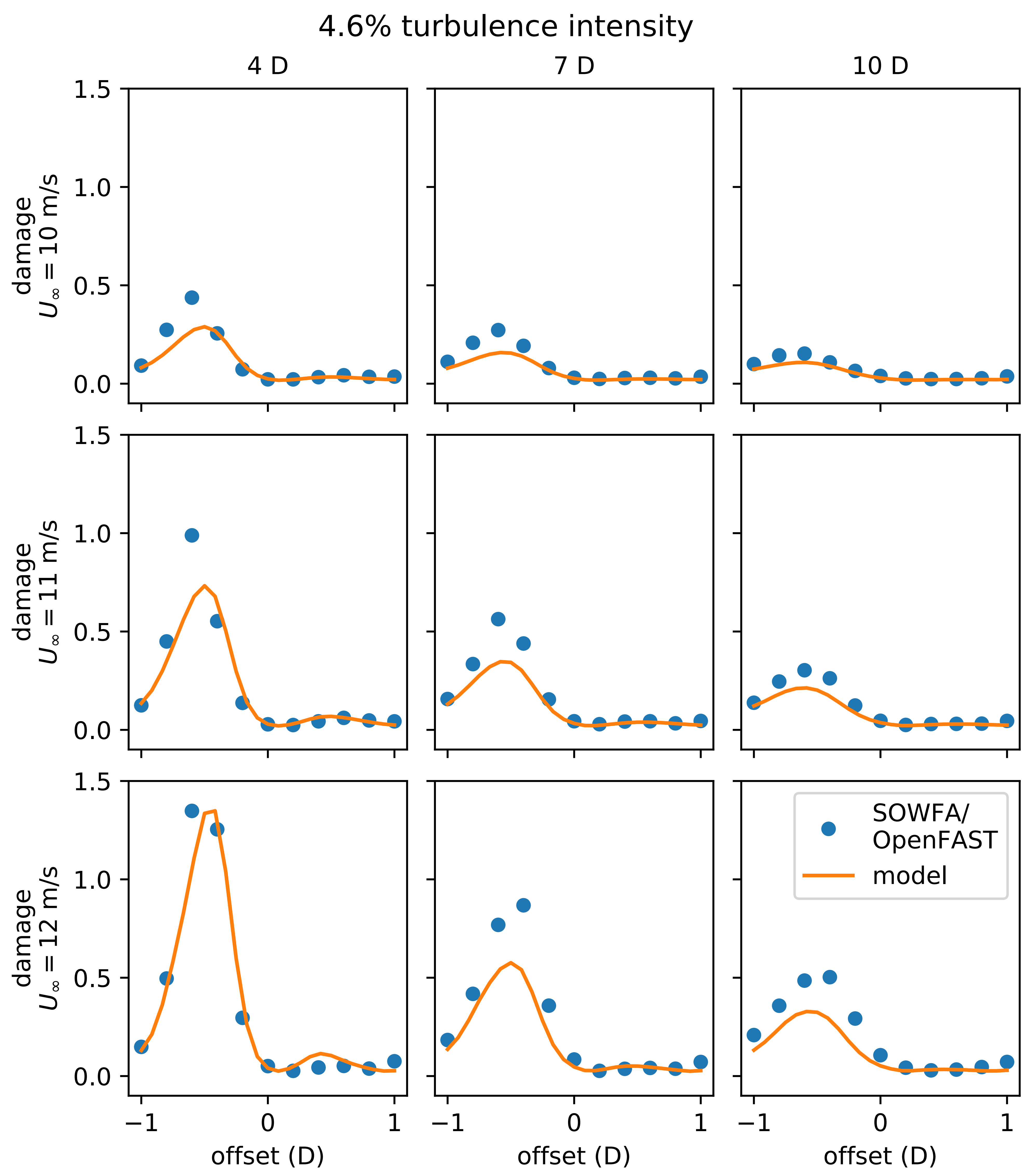

−4
0
4
Δy/d
35
20
Annual Energy Production (GWh)
ξ = 1.0
ξ = 2.0
ξ = 3.0
−4
0
4
Downstream Turbine's
Cross-wind Offset (Δy/d)



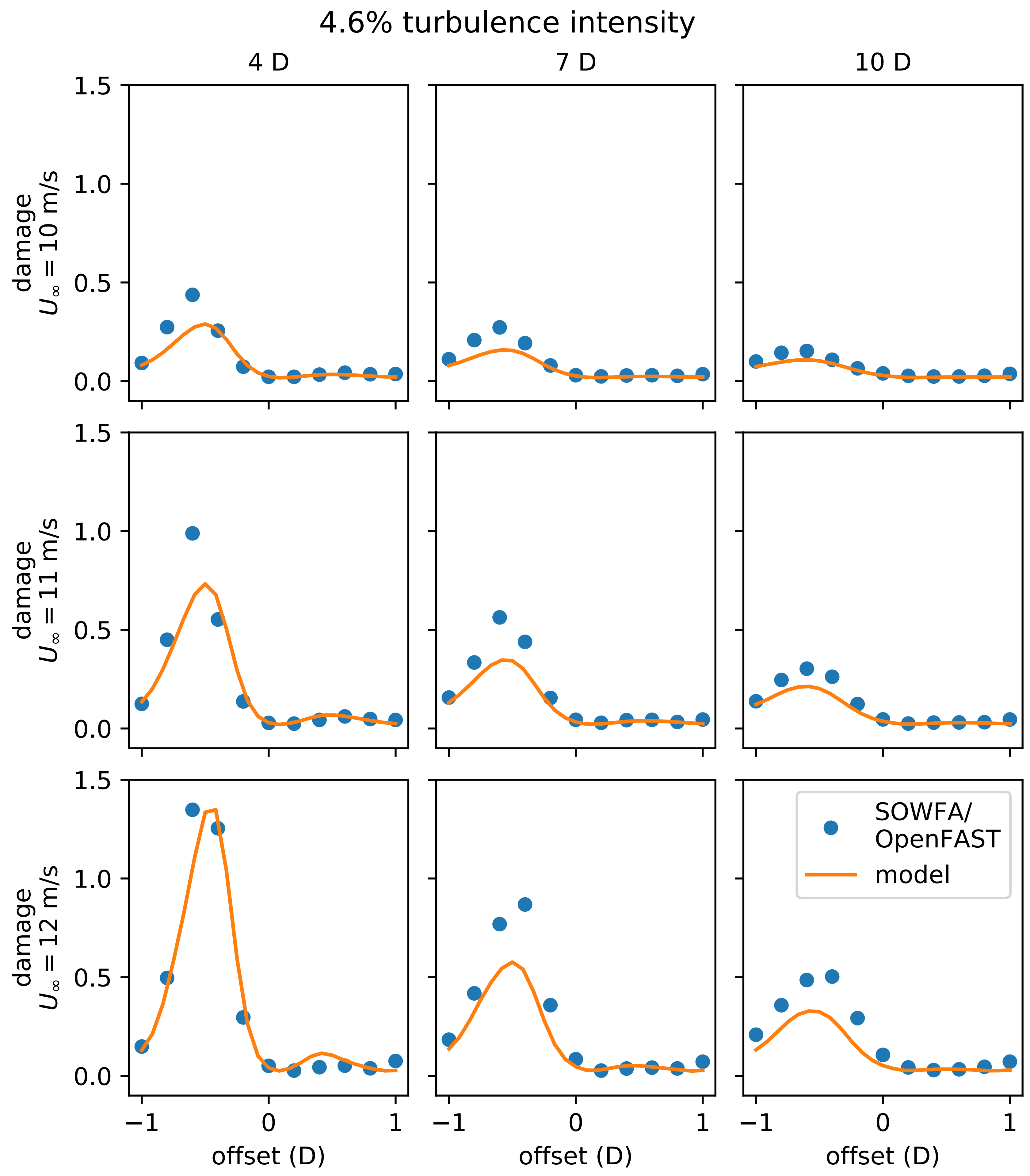
Thomas, J. J., McOmber, S., and Ning, A., "Wake Expansion Continuation: Multi-Modality Reduction in the Wind Farm Layout Optimization Problem," Wind Energy, Vol. 25, No. 4, Apr 2022, pp. 678–699.

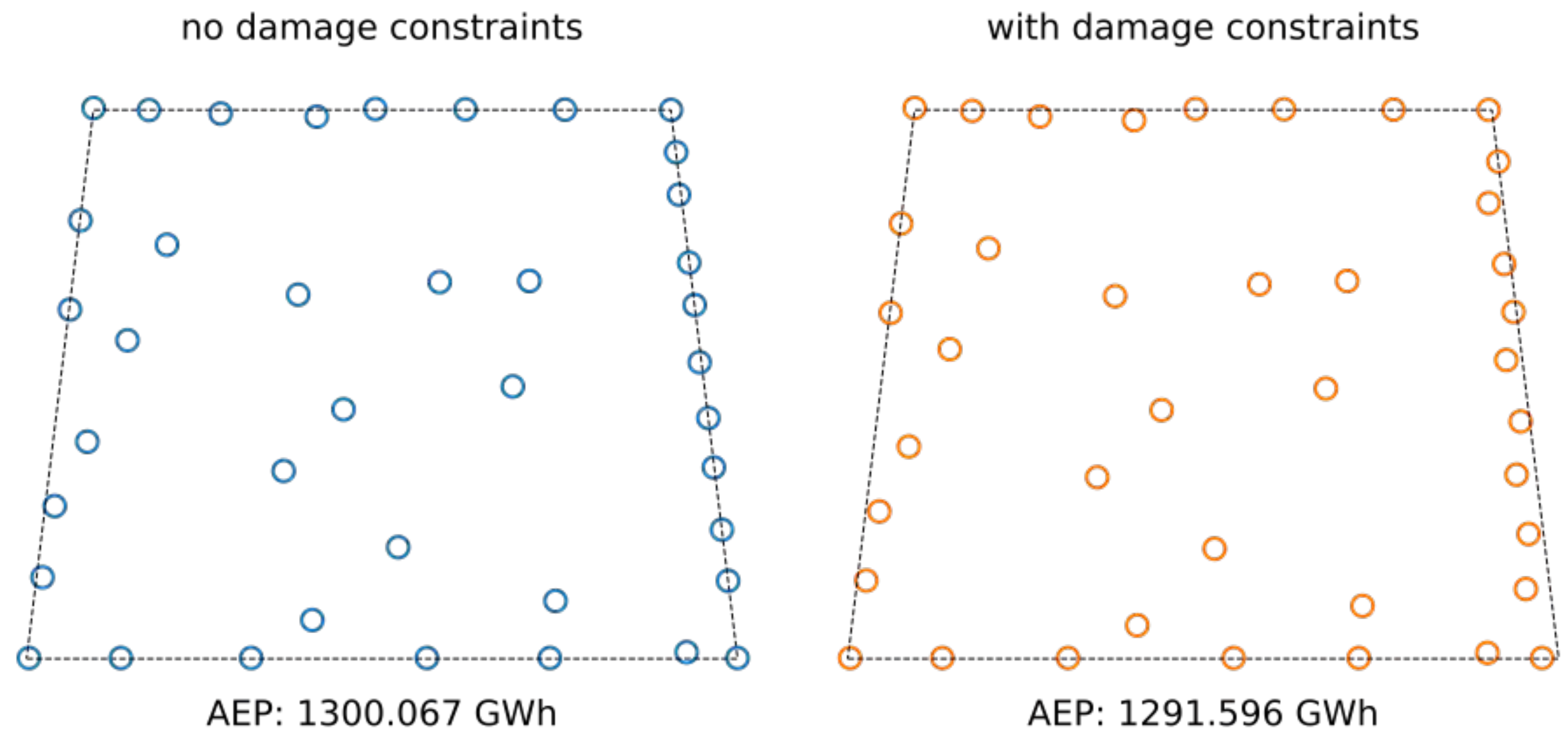
Fatigue Constraints

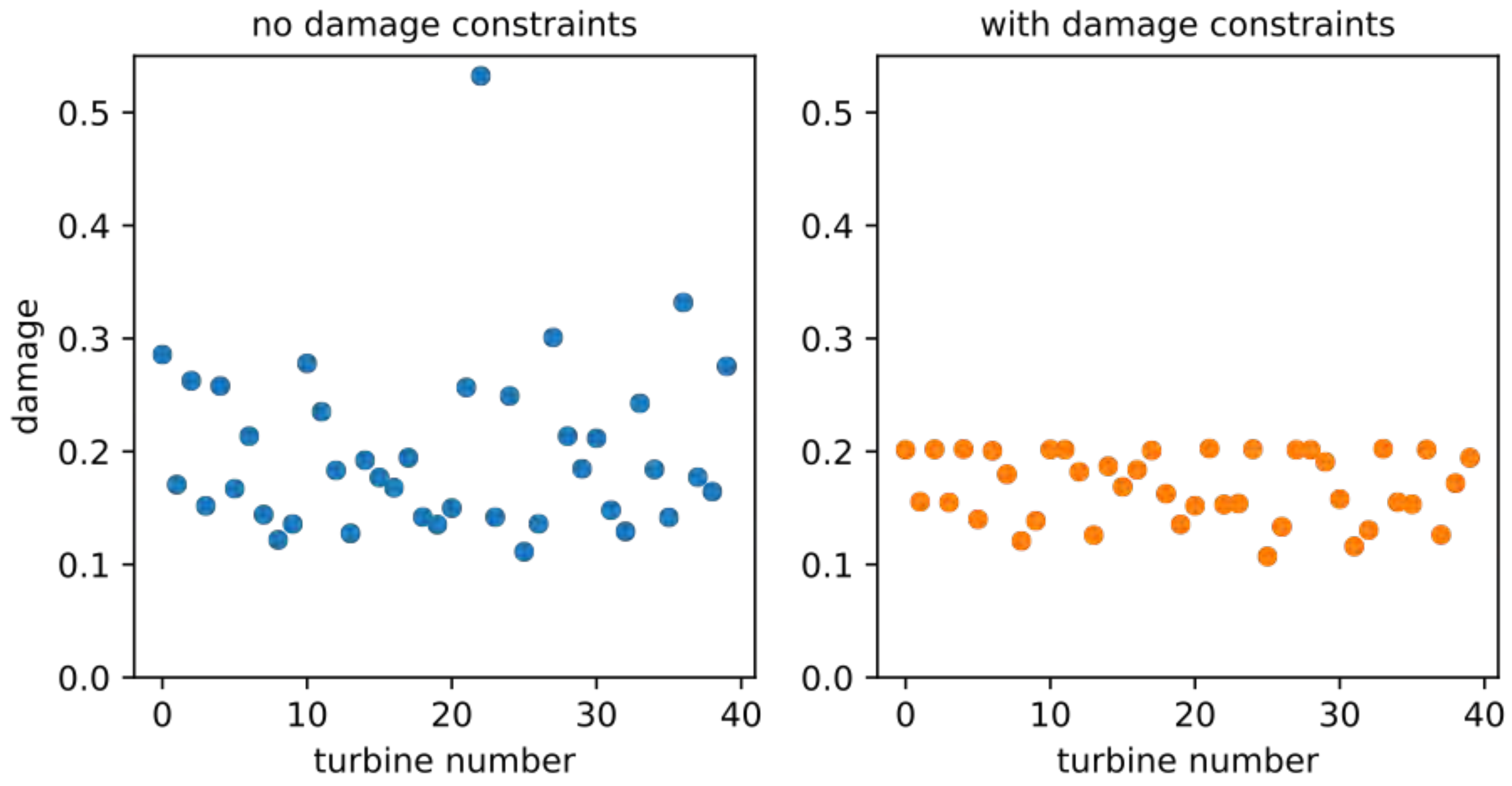




Fatigue Constraints

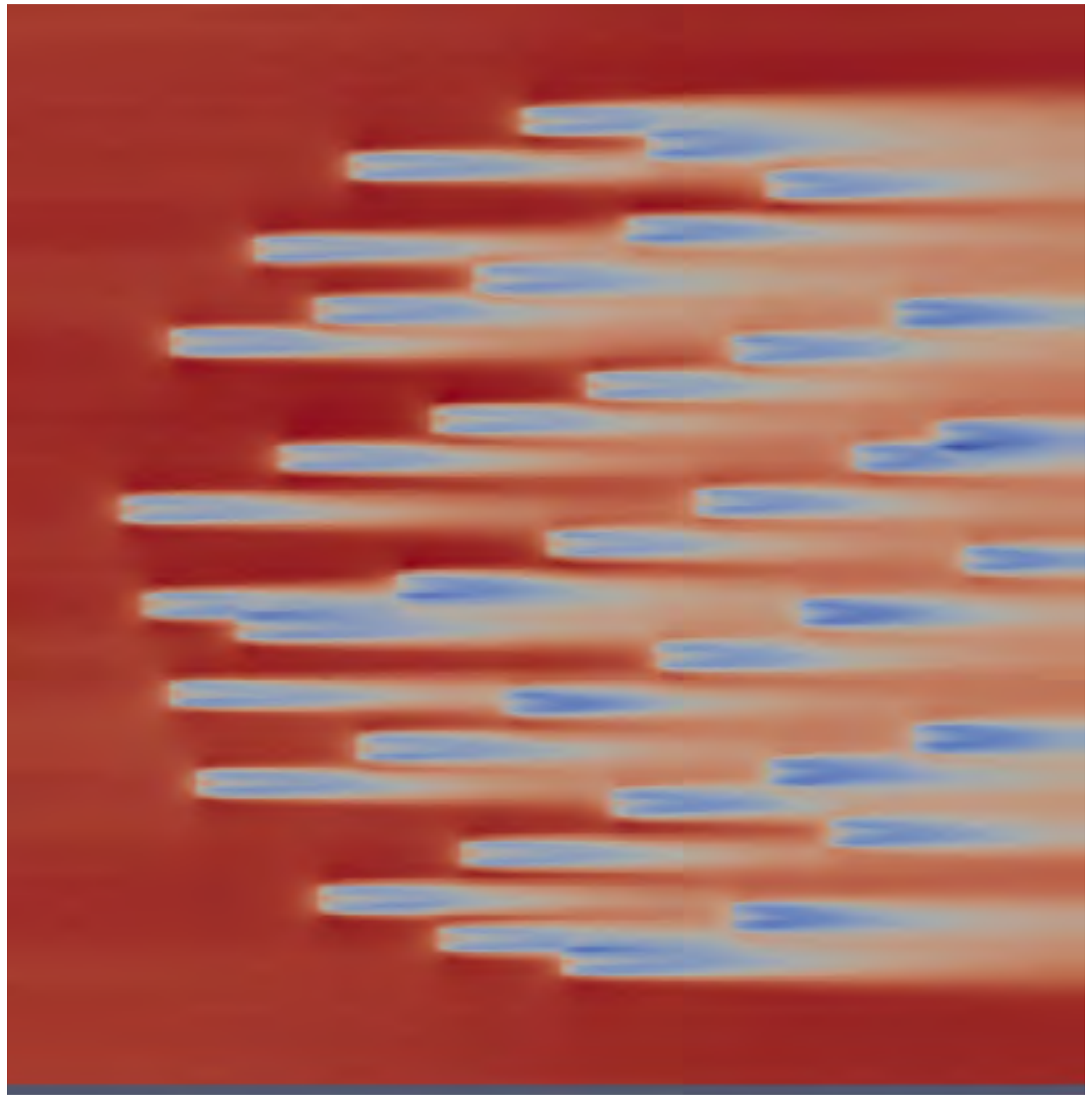


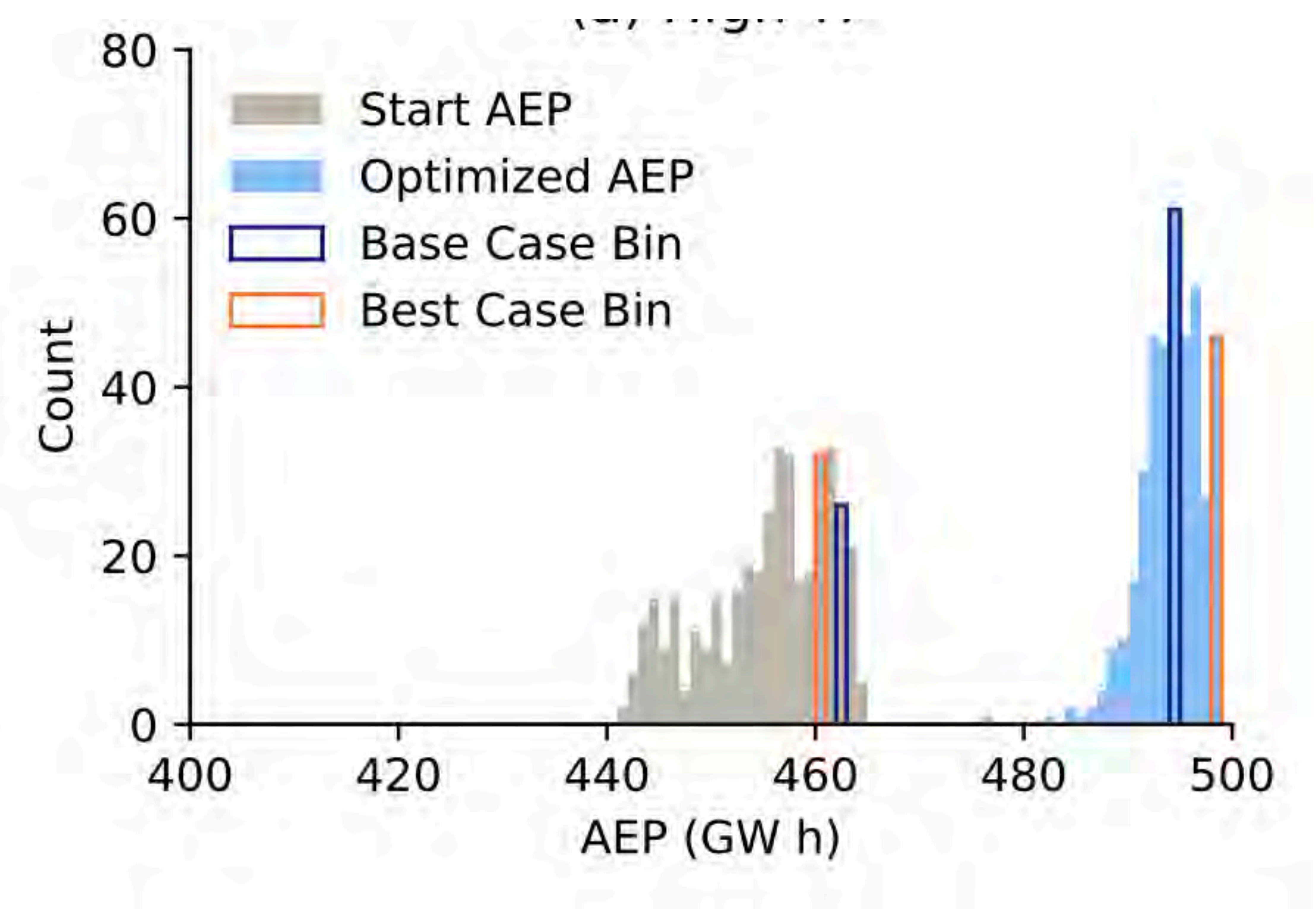


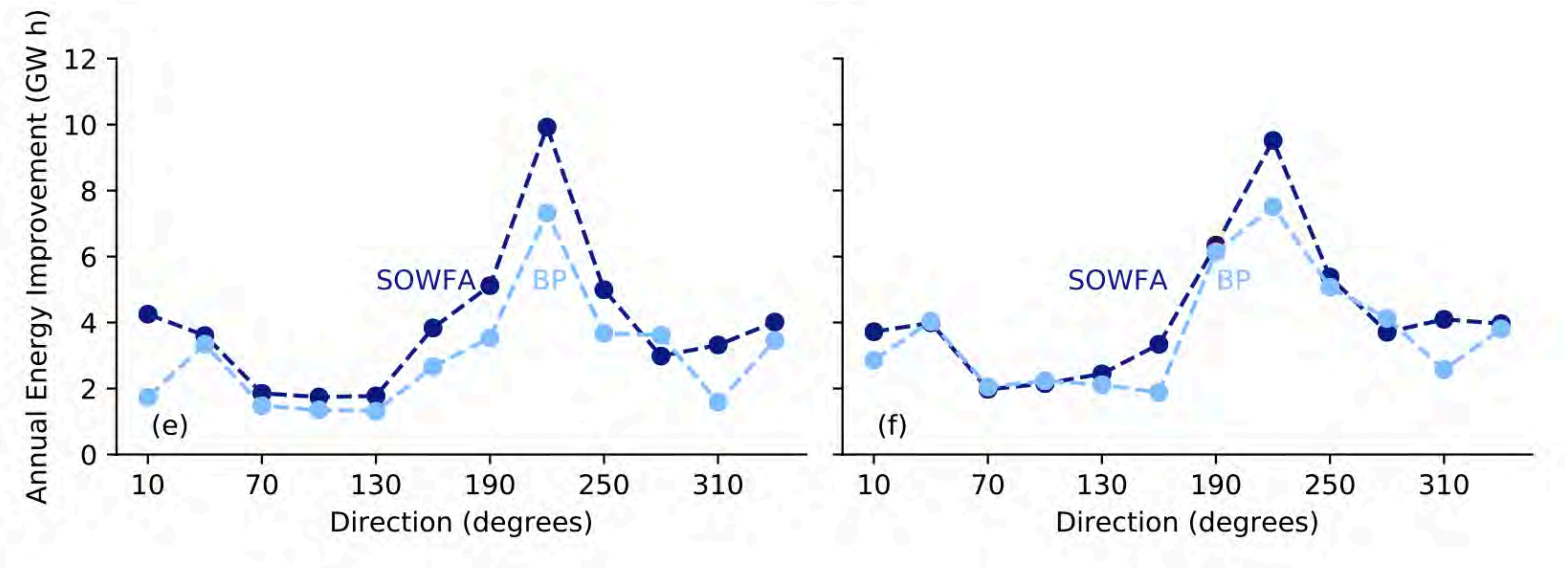


Stanley, A. P., King, J., Bay, C., and Ning, A., "A Model to Calculate Fatigue Damage Caused by Partial Waking during Wind Farm Optimization," Wind Energy Science, Vol. 7, No. 1, Mar 2022, pp. 433–454.

Optimization Comparison to LES

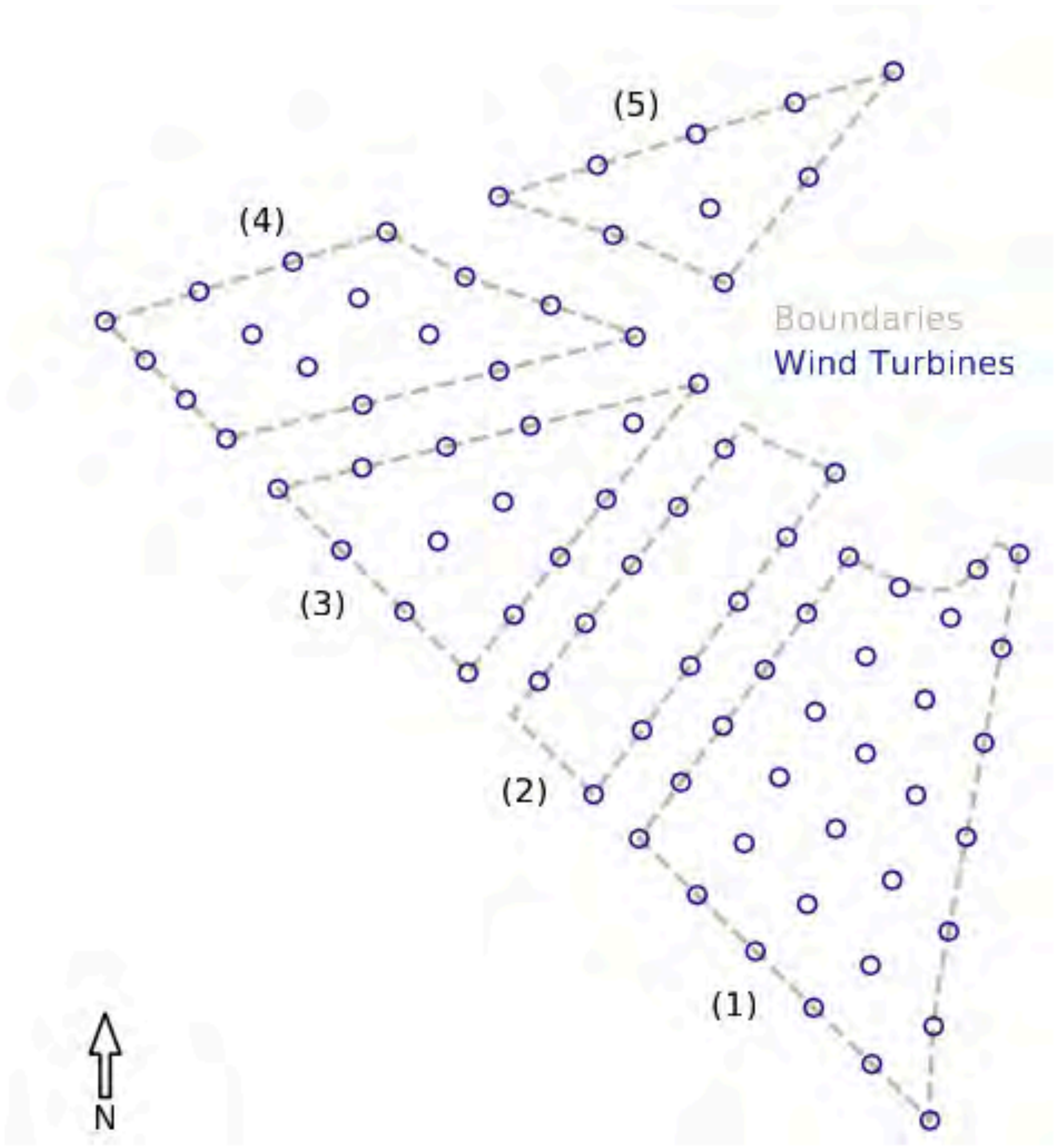


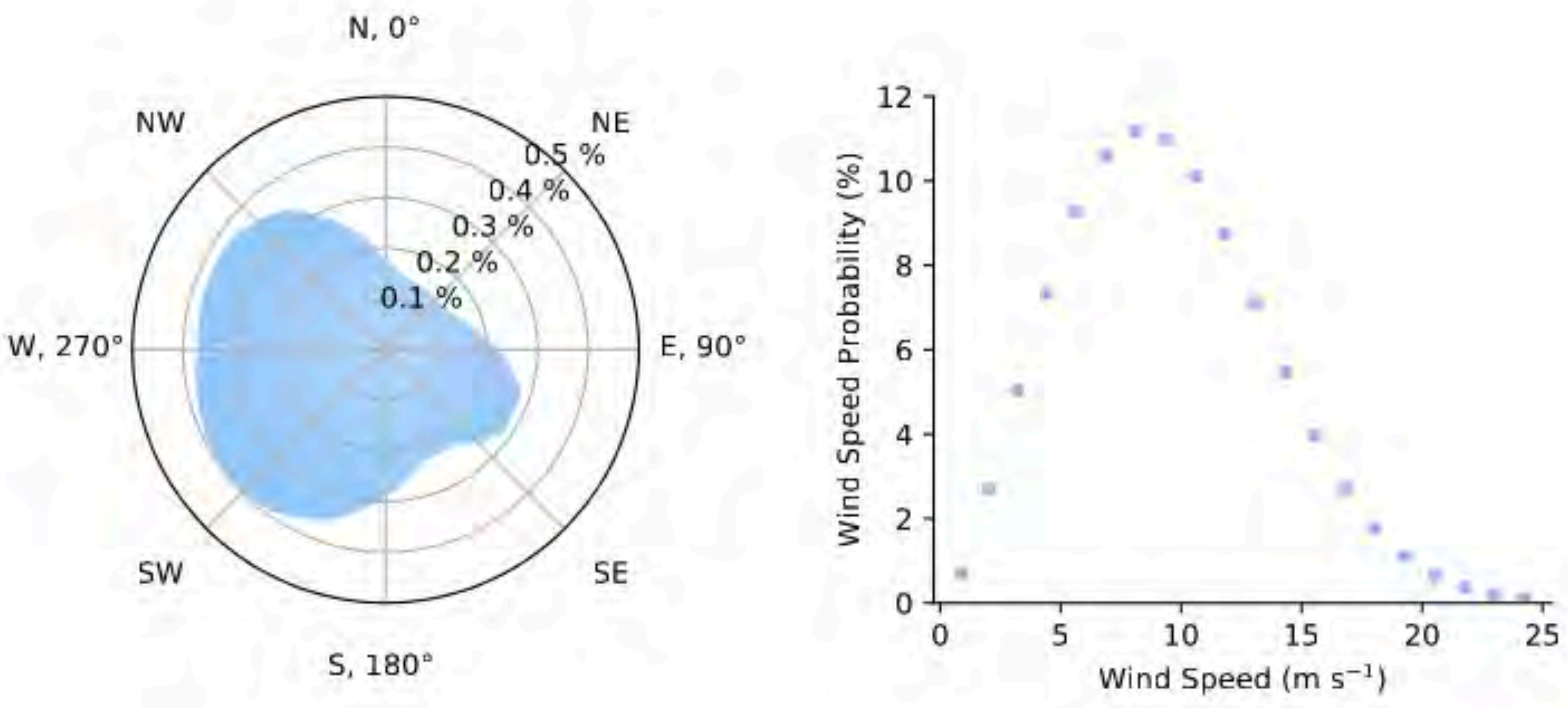




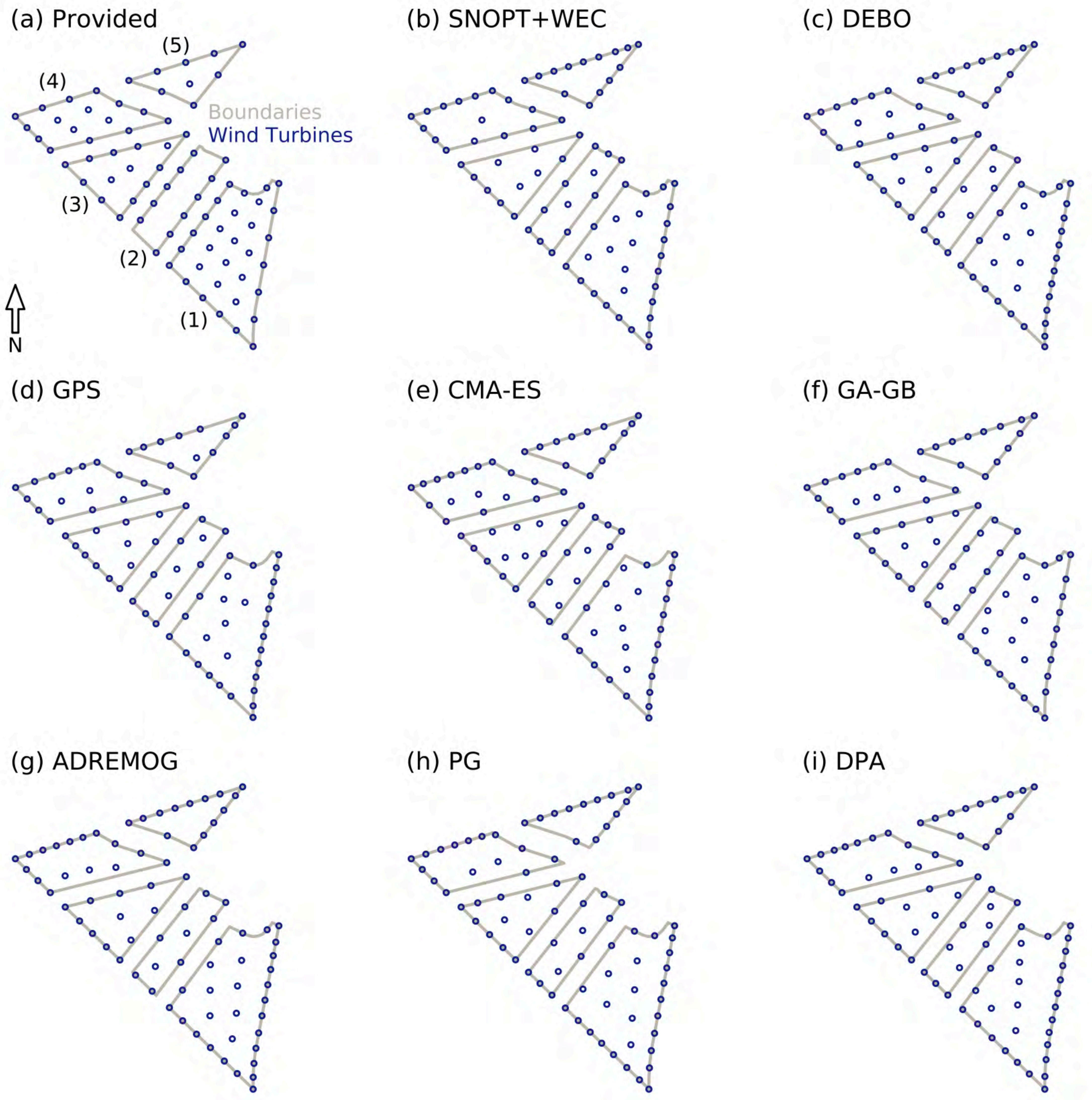
Thomas, J. J., Bay, C. J., Stanley, A. P. J., and Ning, A., "Gradient-Based Wind Farm Layout Optimization Results Compared with Large-Eddy Simulations," Wind Energy Science, Jan 2022, (in review).

Algorithm Comparison for Wind Farm Layout Optimization



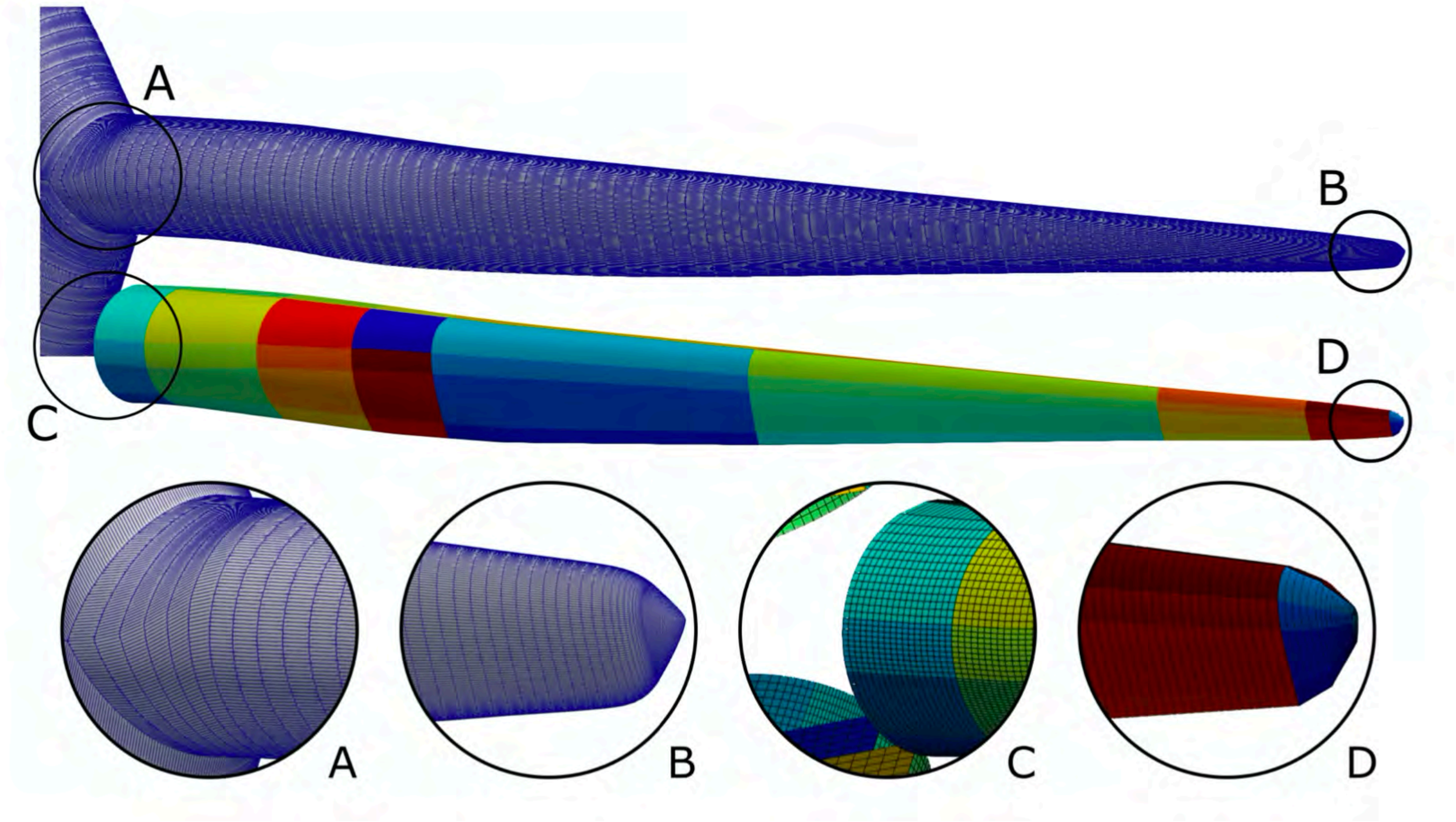


(a) (b)



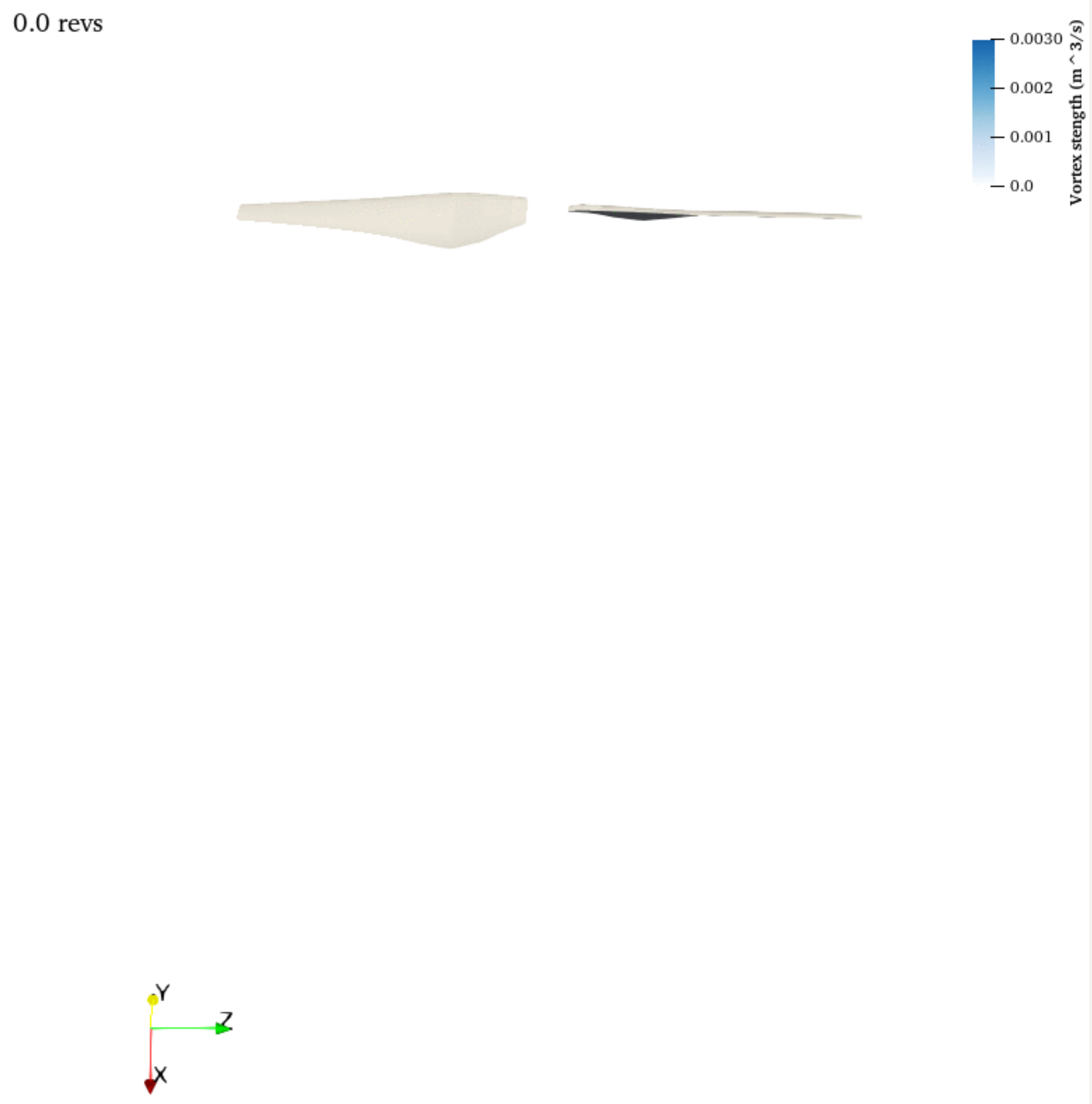
Jared J. Thomas, Nicholas F. Baker, Paul Malisani, Erik Quaeghebeur, Sebastian Sanchez Perez-Moreno, John Jasa, Christopher Bay, Federico Tilli, David Bieniek, Nick Robinson, Andrew P. J. Stanley, Wesley Holt, and Andrew Ning, "A Comparison of Eight Optimization Methods Applied to a Wind Farm Layout Optimization Problem"

Optimization with Higher Fidelity Analysis Tools



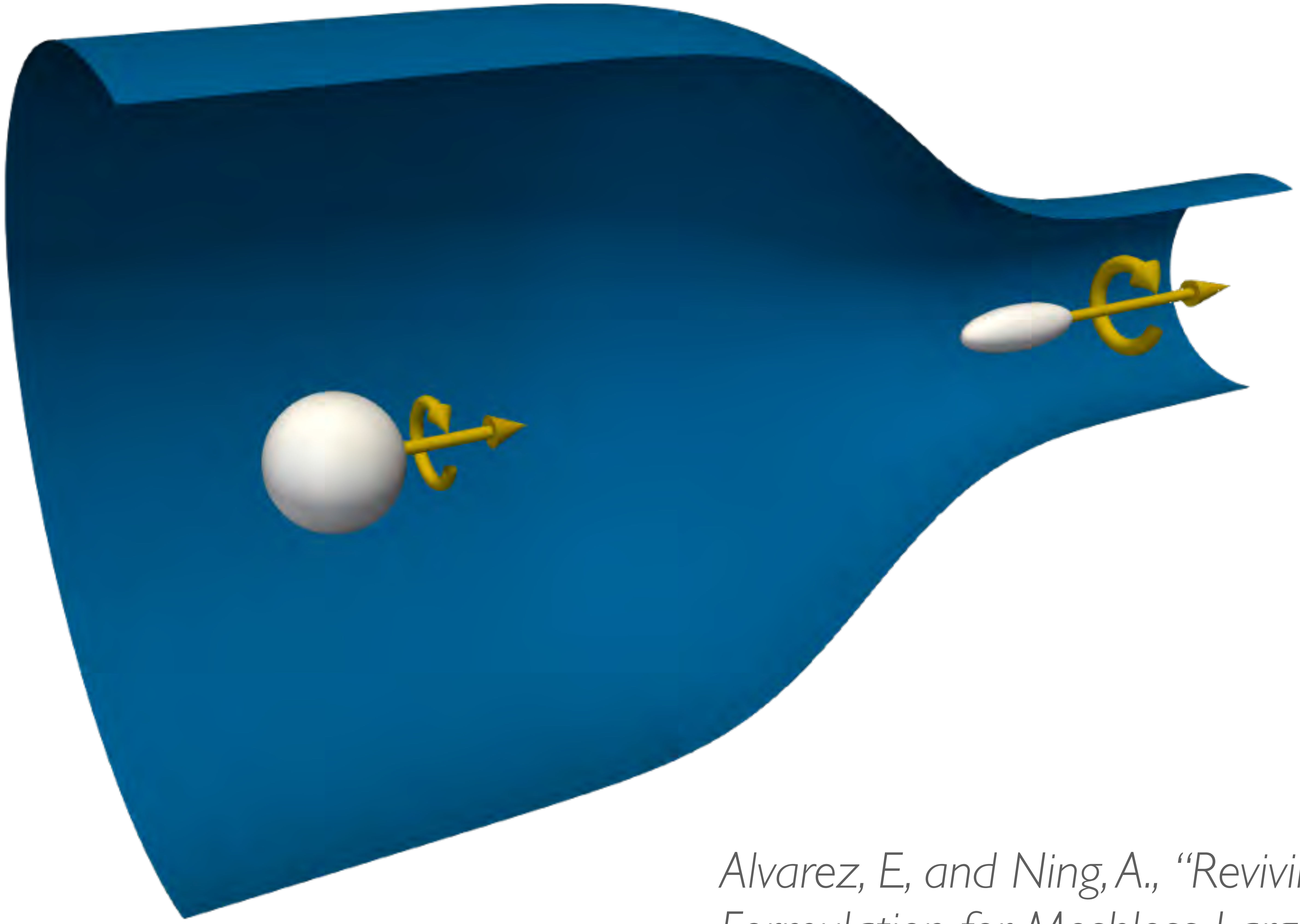
Caprace, D.-G., Cardoza, A., Ning, A., Mangano, M., He, S., and Martins, J. R. R. A., "Incorporating High- Fidelity Aerostructural Analyses in Wind Turbine Rotor Optimization," AIAA SCITECH 2022 Forum, San Diego, CA, Jan 2022.

Vortex particle methods



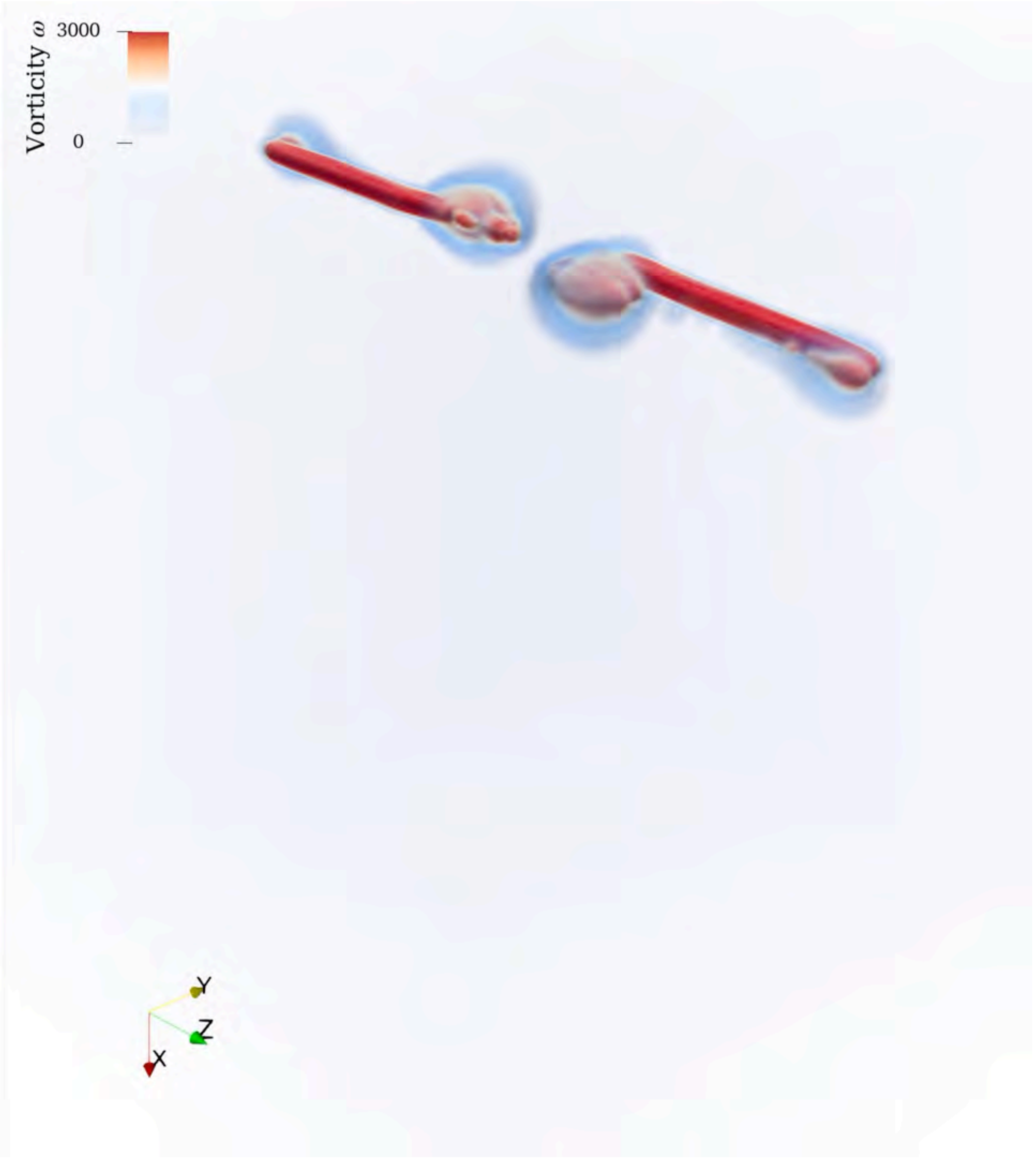
Classic VPM

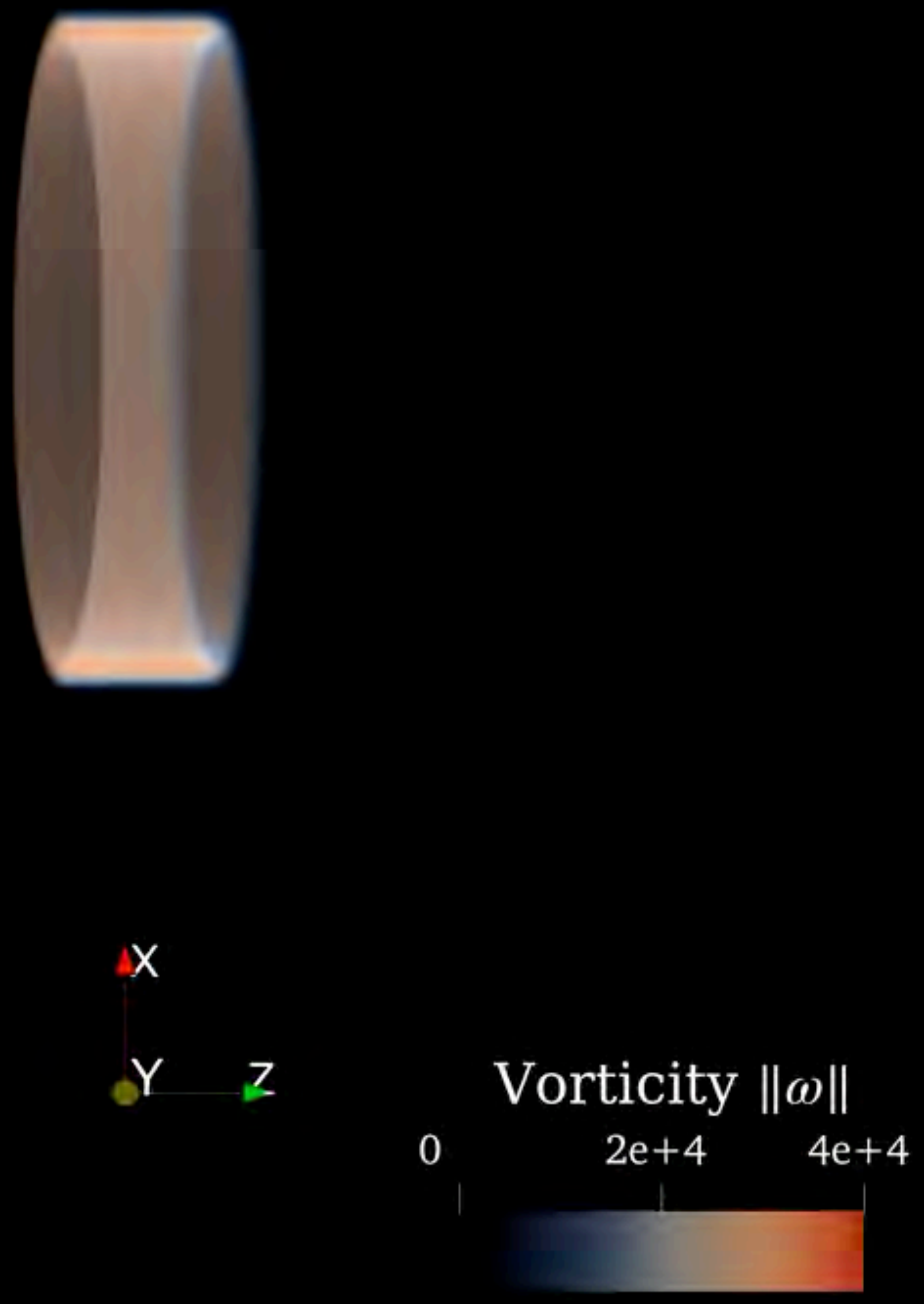
We have derived a new *stable* set of governing equations

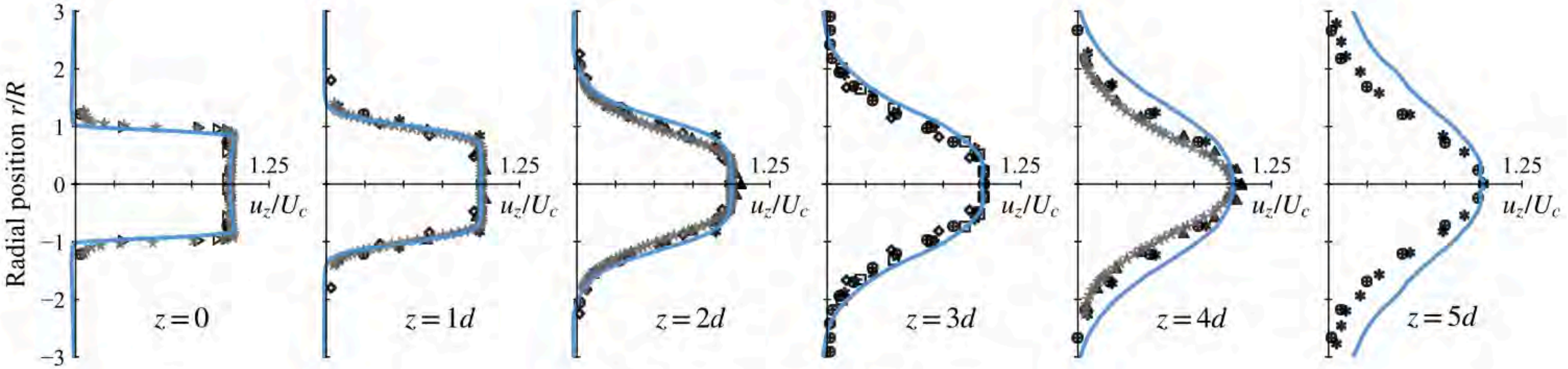


Alvarez, E, and Ning, A., "Reviving the Vortex Particle Method: A Stable Formulation for Meshless Large Eddy Simulation," 2022, (in review)

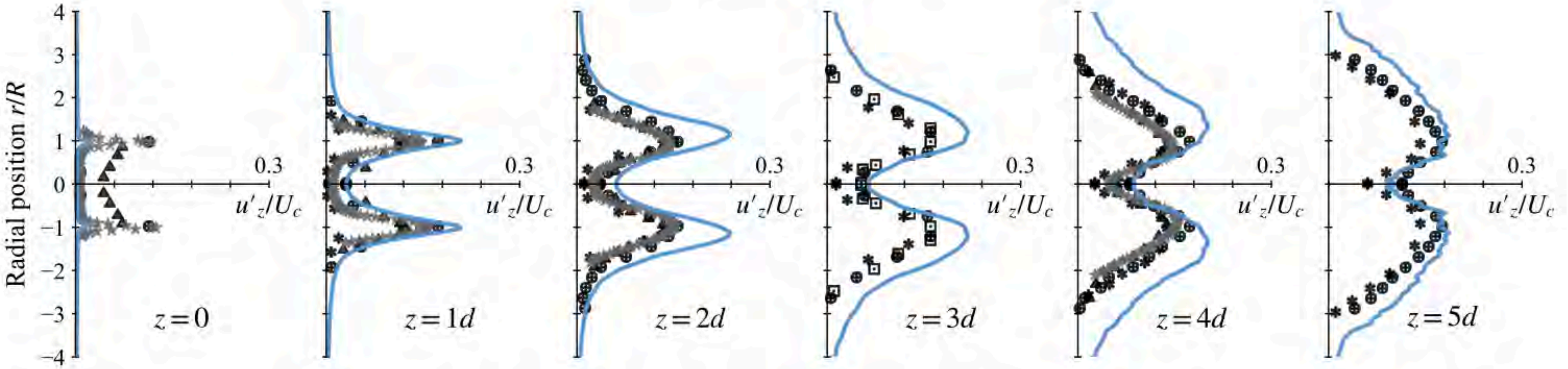

SFS $|C_d|$
0.001
0.01
0.1
1
Vortex strength Γ
0
4e-05
Y
Z
X


Vorticity ω
3000
0
Y
Z
X

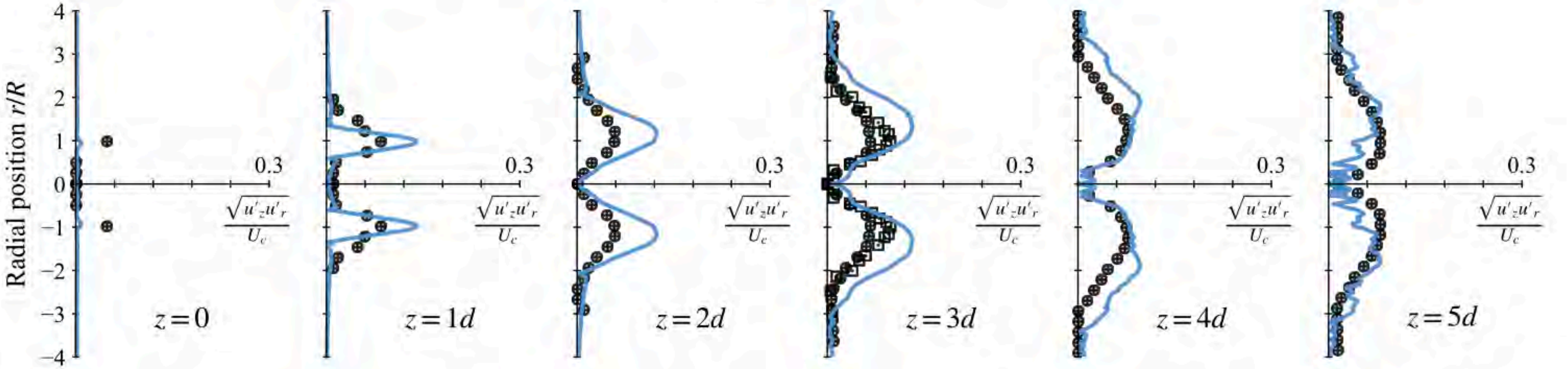

X
Y
Z
Vorticity ||ω||
0
2e+4
4e+4



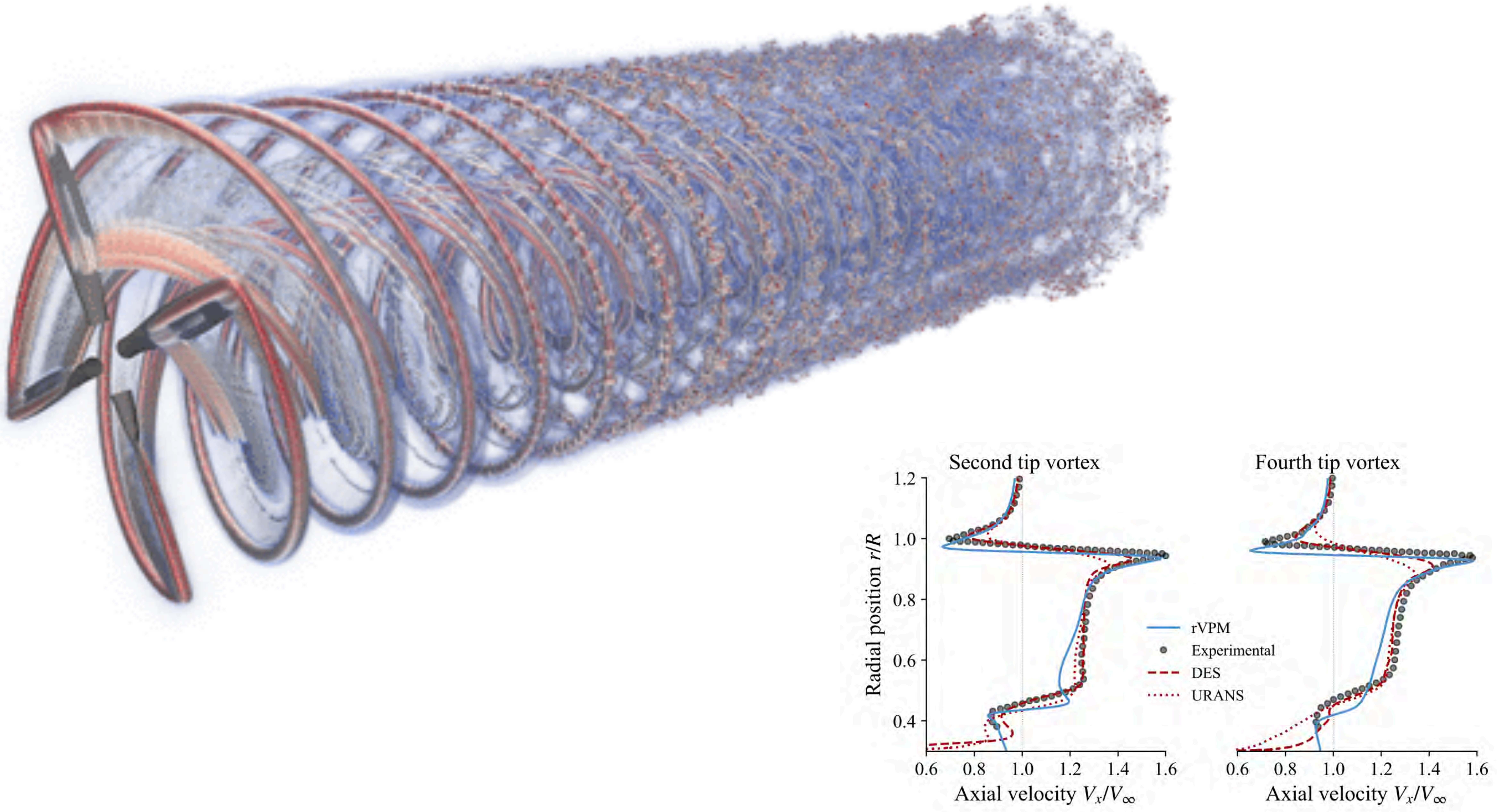
(a) Mean streamwise velocity.



(b) Fluctuating component of the streamwise velocity.



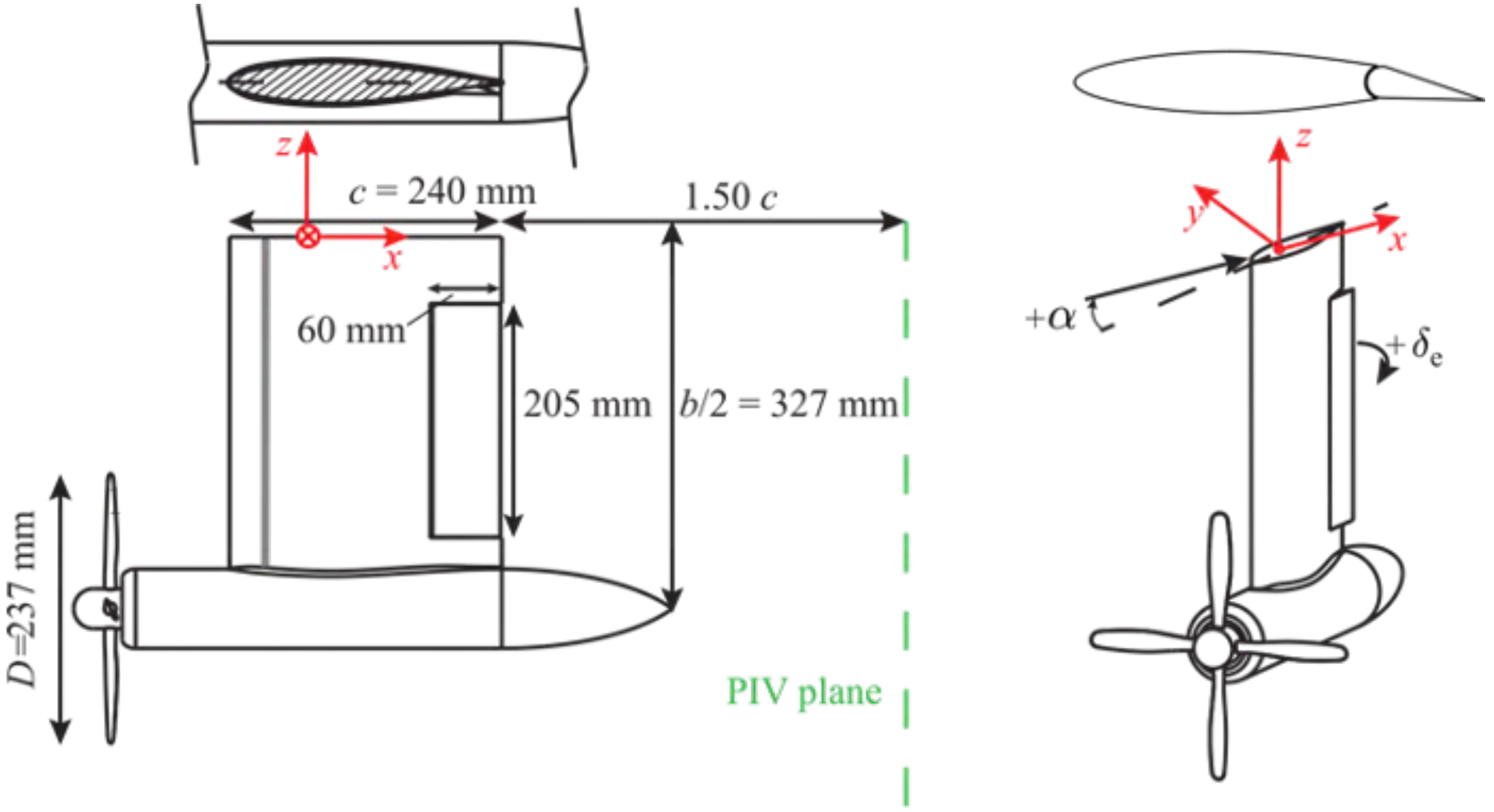
(c) Reynolds stress between streamwise and radial fluctuating components.



Second tip vortex
Fourth tip vortex
Radial position r/R
Axial velocity V_x/V_∞
Axial velocity V_x/V_∞
rVPM
Experimental
DES
URANS
1.2
1.0
0.8
0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6

Simulation	**CPU Cores**	**Wall-Clock**	**Core-Hours**	**rVPM Speedup**
rVPM	32	4.3 hours	140	–
URANS	192	9.6 hours	1.8k	~10x faster
LES-ALM	845	24 hours	20k	~100x faster
LES-IBM	1000	96 hours	96k	~500x faster
DES	1008	300 hours	300k	~1000x faster

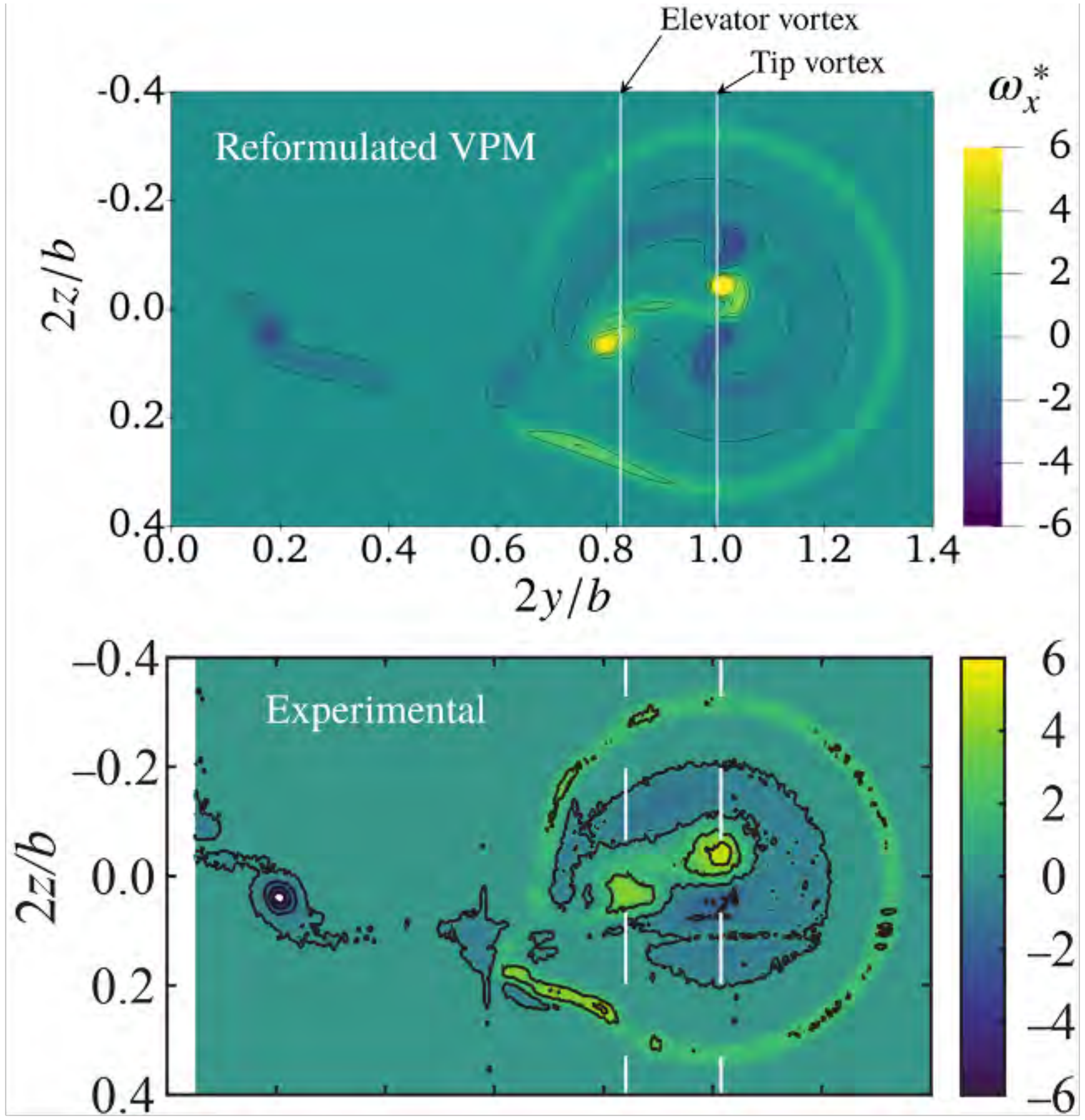
Tip-mounted Propeller



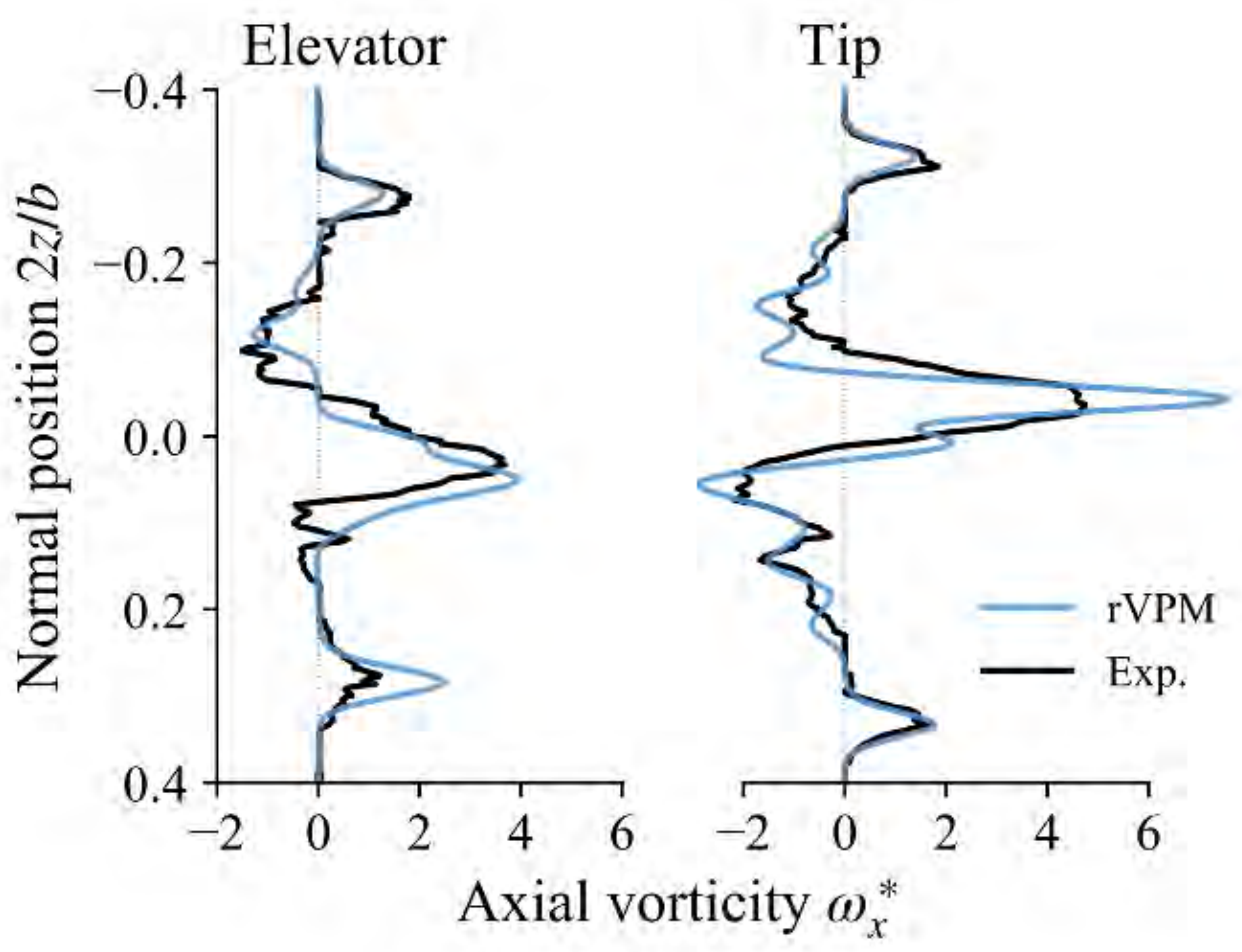
Experiment: van Arnhem, "Unconventional Propeller-Airframe Integration for Transport Aircraft Configurations," Ph.D. thesis, 2022

Test 1: Elevator deflection

$(\alpha = 0°,\ \delta_e = +10°)$

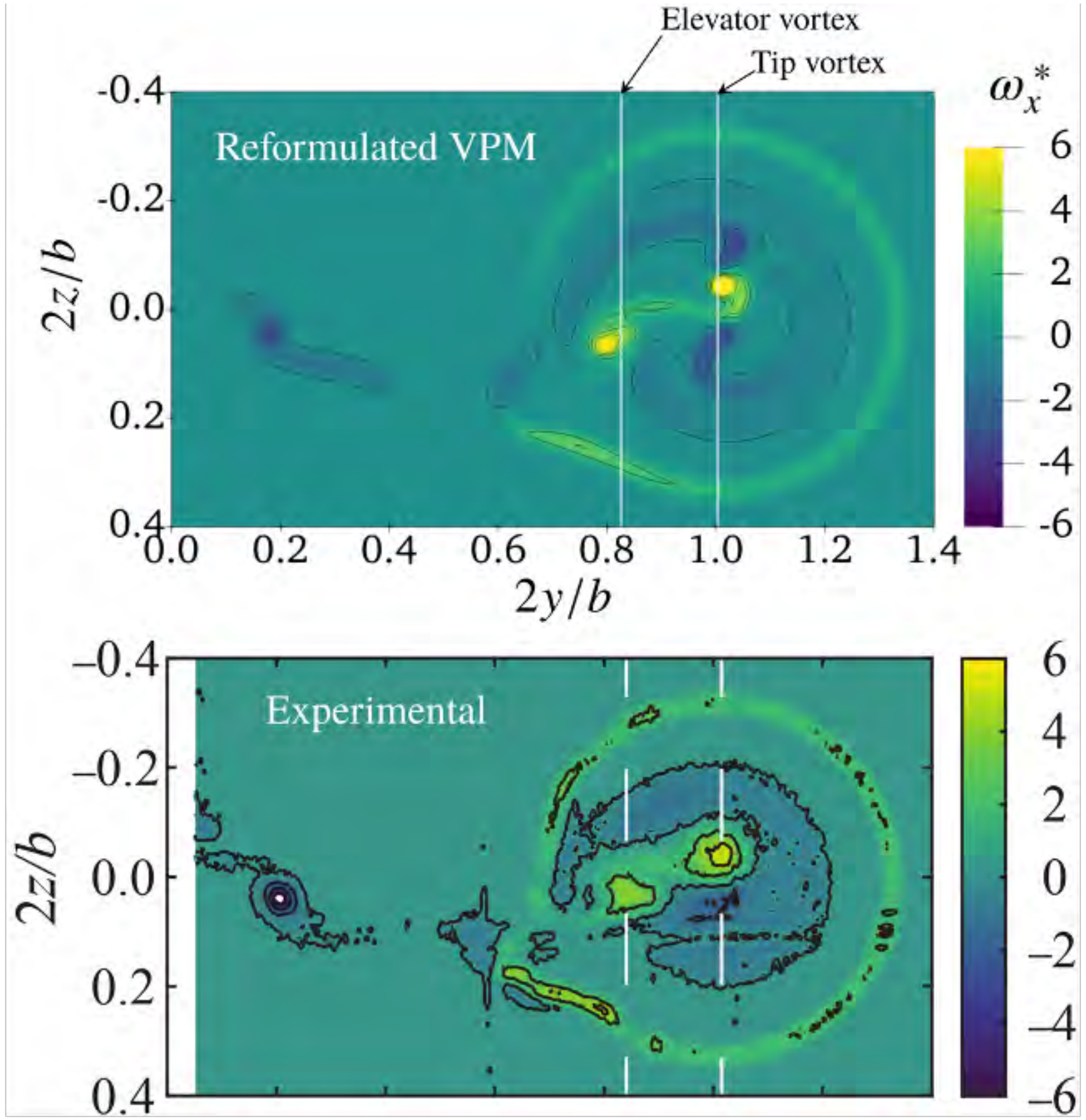


Tip-mounted Propeller



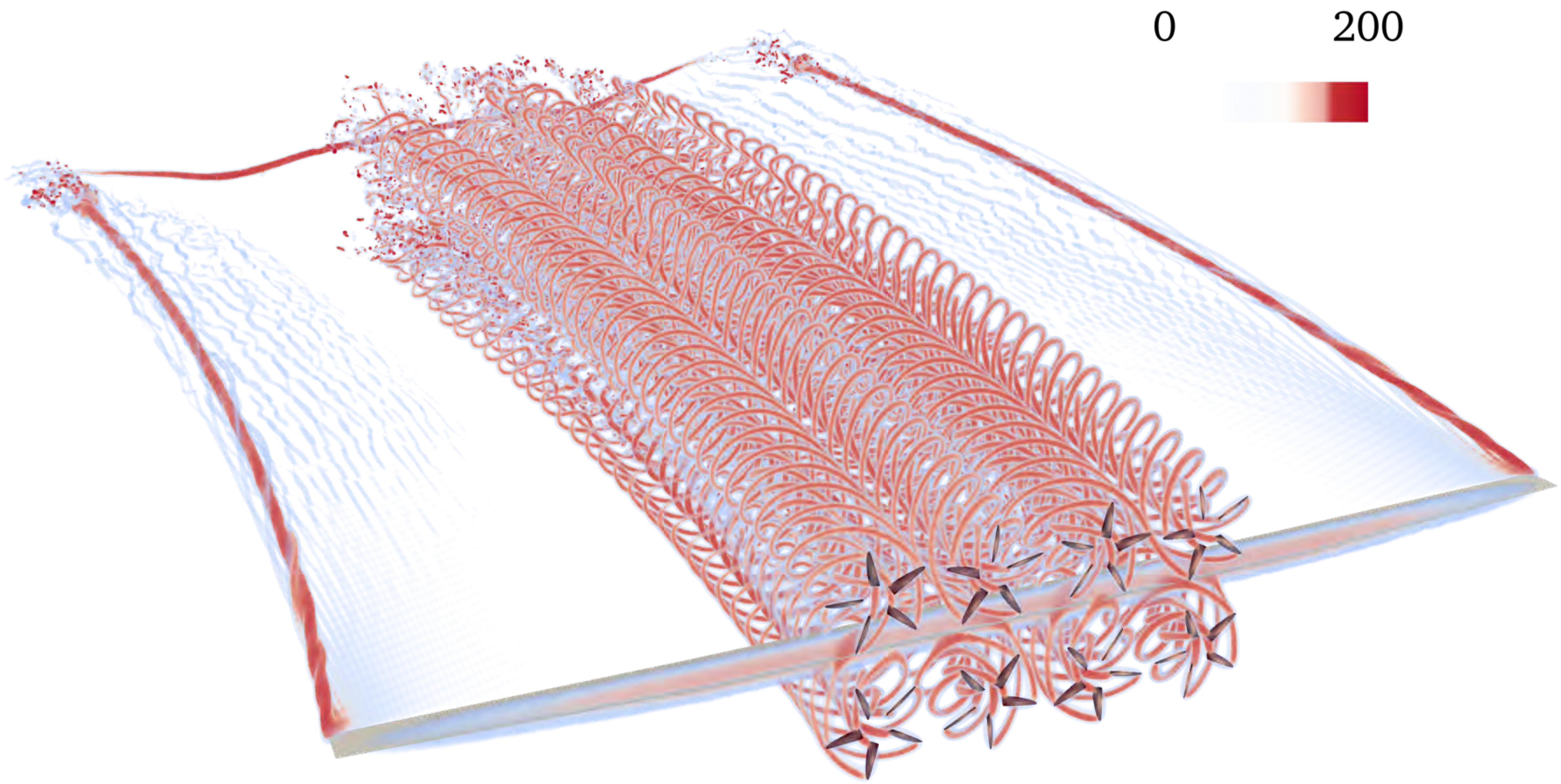
Test 1: Elevator deflection

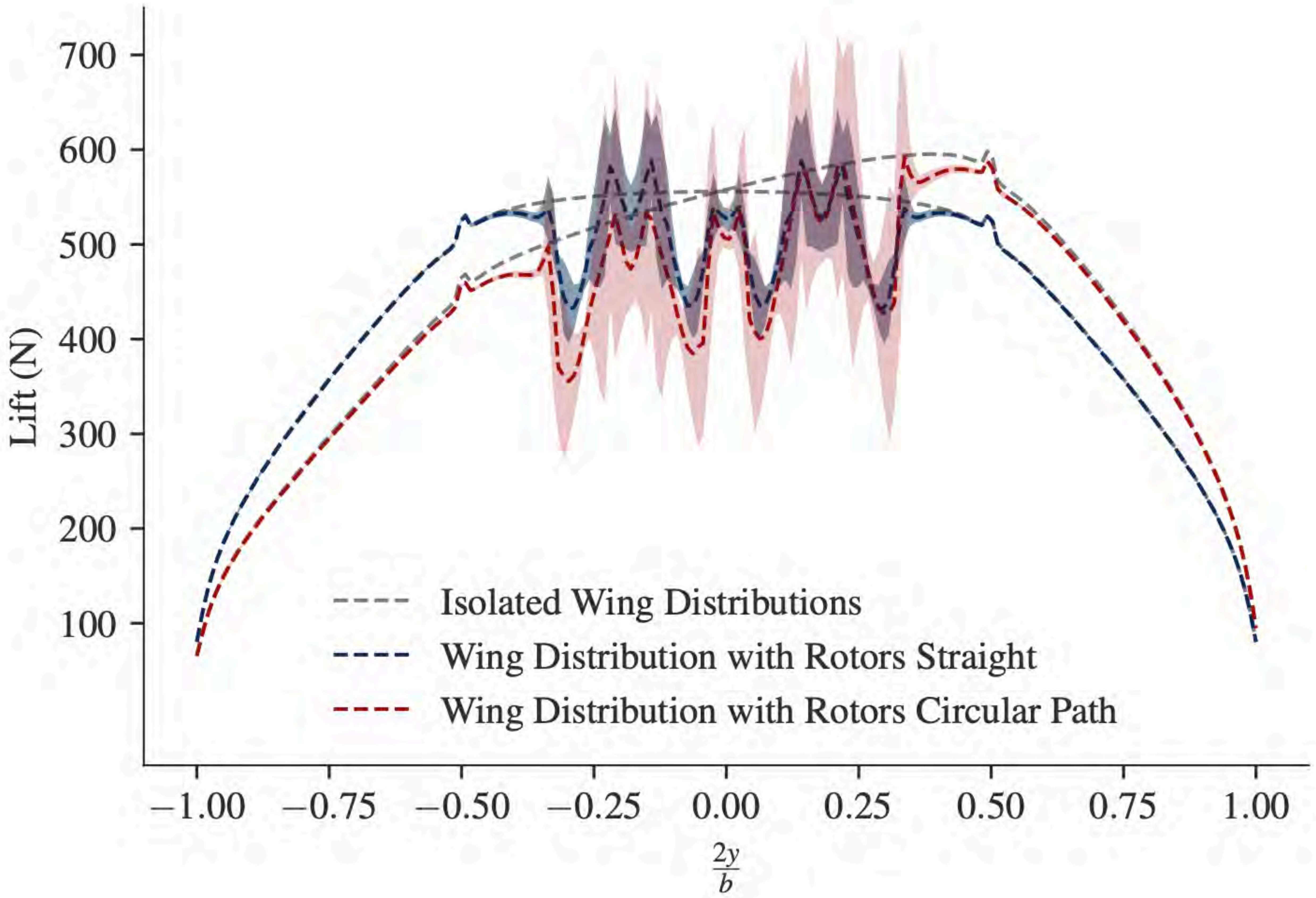
$(\alpha = 0°,\ \delta_e = +10°)$



Z
X
Y
70
50
30
2
Vorticity (1/s)

Airborne Wind





Mehr, J., Alvarez, E. J., and Ning, A., "Interactional Aerodynamics Analysis of a Multi-Rotor Energy Kite," Wind Energy, Apr 2022, (In review).

Discrete Adjoint

N x N Brusselator stiff reaction-diffusion PDE

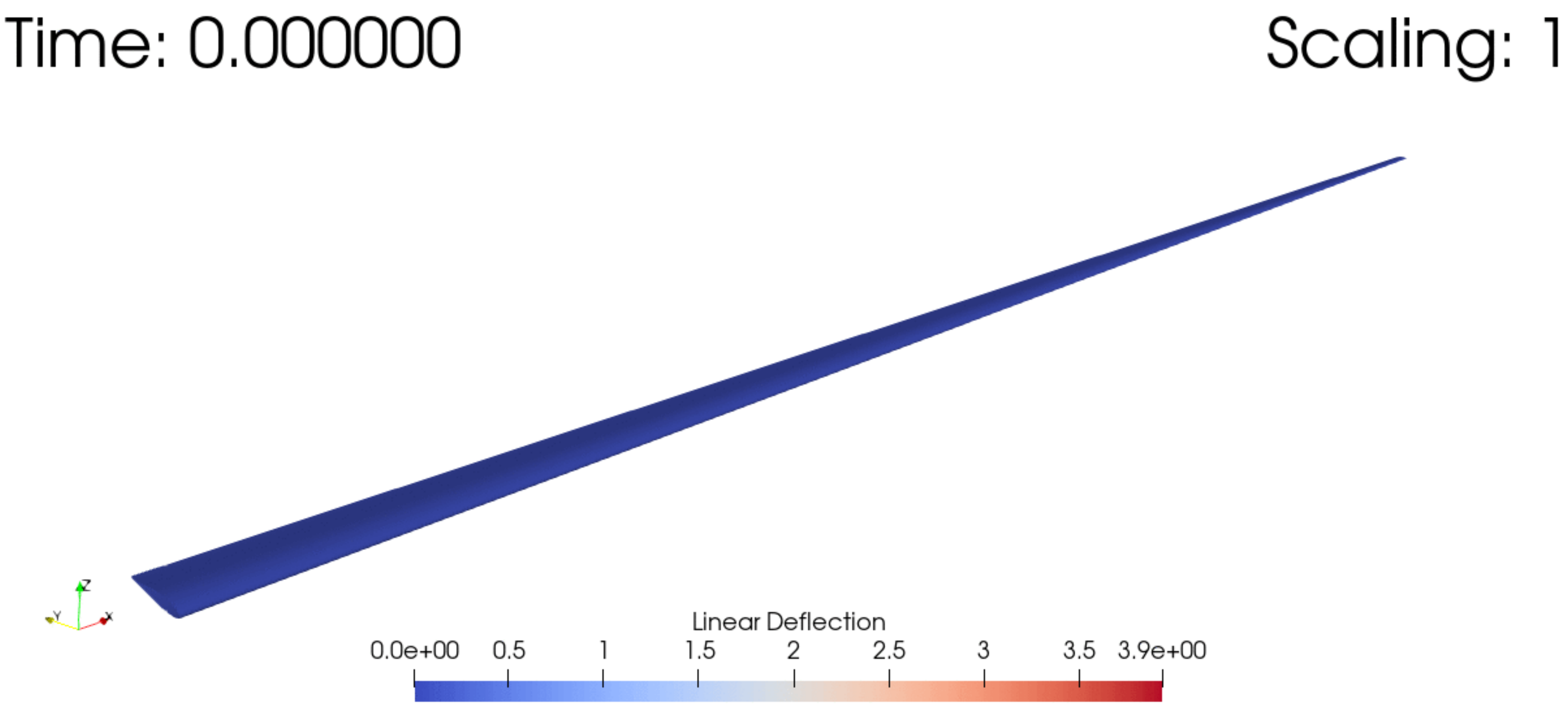
$$\frac{\partial u}{\partial t} = p_2 + u^2 v - (p_1 + 1)u + p_3(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) + f(x, y, t)$$
$$\frac{\partial v}{\partial t} = p_1 u - u^2 v + p_4(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$$

$$f(x, y, t) = \begin{cases} 5 & \text{if } (x - 0.3)^2 + (y - 0.6)^2 \leq 0.1^2 \text{ and } t \geq 1.1 \\ 0 & \text{else} \end{cases}$$

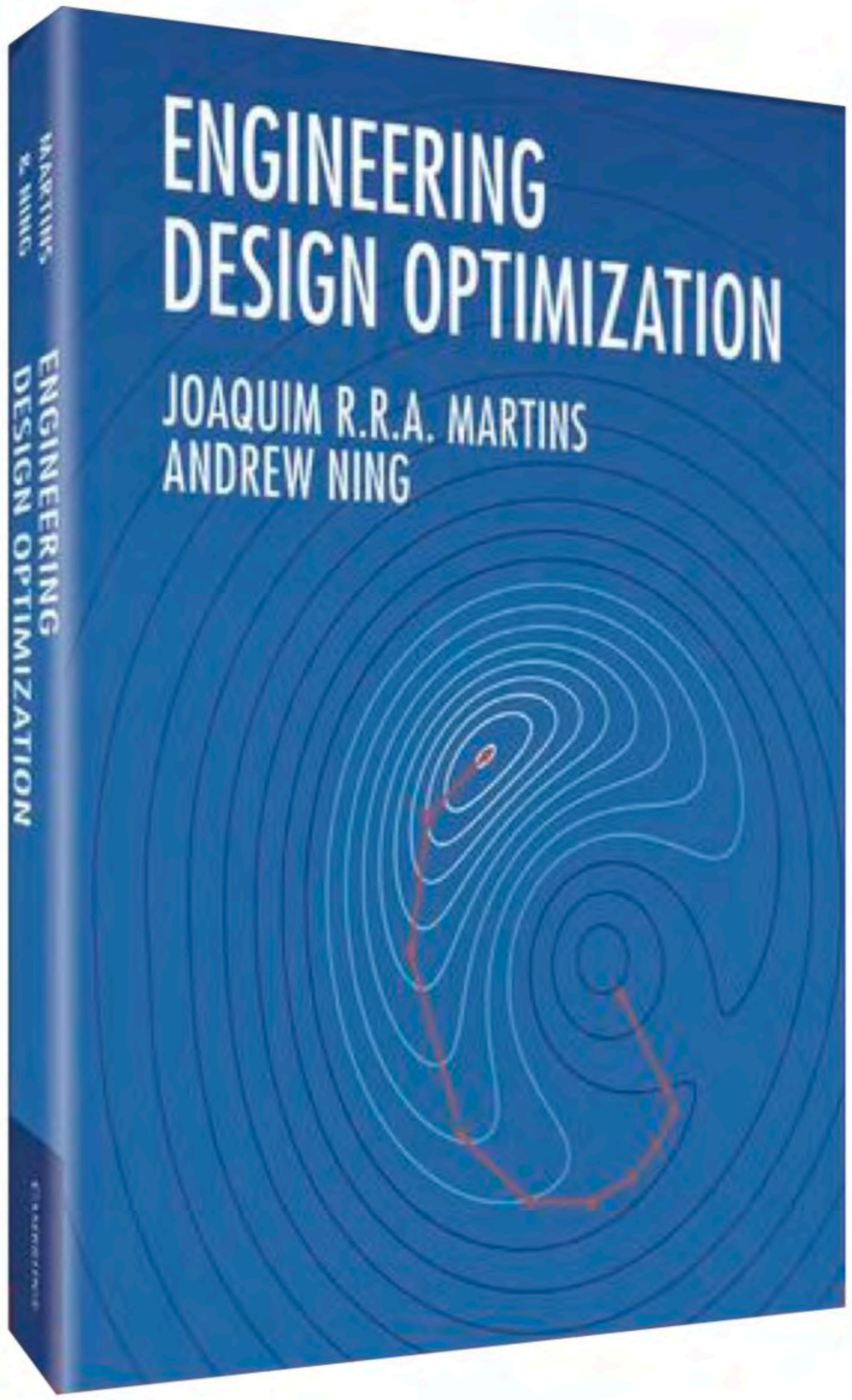
Discrete Adjoint

Method	Time (ms)
Forward AD	800
Reverse AD	1040
Continuous Direct	3601
Continuous Adjoint	554
Discrete Adjoint (Forward VJP)	680
Discrete Adjoint (Reverse VJP)	61

GXBeam



McDonnell, T. and Ning, A., Geometrically Exact Beam Theory for Gradient-Based Optimization, (in preparation)



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FLOW Lab

https://flow.byu.edu

CCBlade.jl

GXBeam.jl

DiscreteAdjoint.jl

FLOWFarm.jl

SNOW.jl

FLOWUnsteady.jl / FLOWVPM.jl

FLOWNoise.jl

NREL NATIONAL RENEWABLE ENERGY LABORATORY

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