

An aerial photograph of a city, likely Zurich, showing a river with a dam or bridge structure, surrounded by dense urban buildings and greenery. The image is partially obscured by a blue text box.

# A Feedback-Optimization Approach to Resilient Power System Operation

**Saverio Bolognani**

NREL Workshop on Resilient Autonomous Energy Systems

# UNICORN project

A Unified Control Framework for Real-Time Power System operation



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Le réseau  
de transport  
d'électricité



Schweizerische Eidgenossenschaft  
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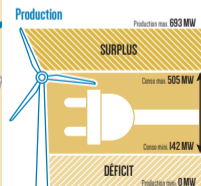
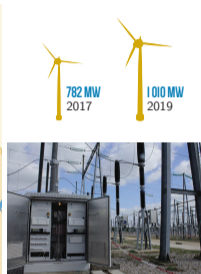
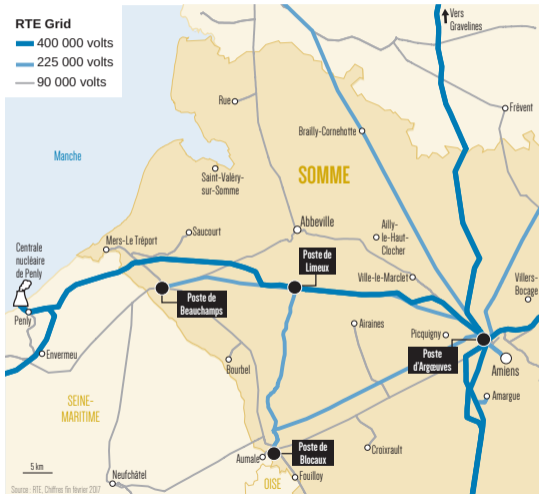
# Outline

1. Real-time power system operation
2. Feedback optimization design
3. Numerical experiments: French subtransmission grid
4. Feedback optimization for a resilient power grid
5. Conclusions

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# A more responsive grid is needed

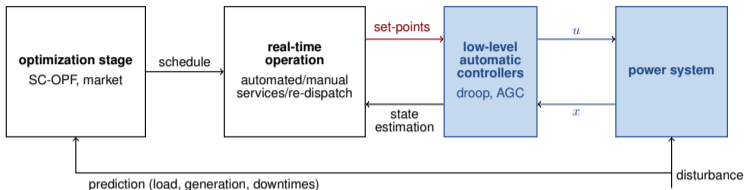


- Larger share of uncontrollable generation
- Distributed generation
- Voltage and line flow constraints

## Future real-time operation

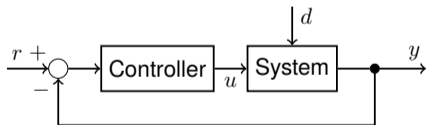
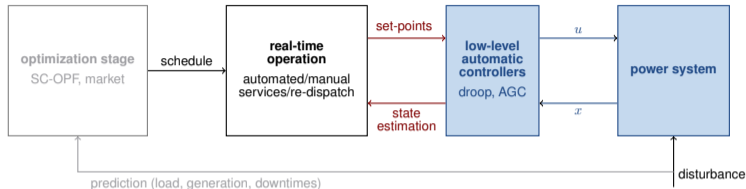
- Online monitoring and measurement
- Real-time operational specifications
- Responsive to fast disturbances

# Available actuation (=set-points)



- **Active power curtailment**
  - ramp up/down limits (inverters: 0 s, wind: 20 s in emergency, 60 s otherwise)
- **AVR (Automatic Voltage Regulators) set-points**
  - example: in France, remotely adjusted every 10 s
- **Active power injection from storage**
  - Minimal delay, high flexibility
- **Reactive power injection**
  - any inverter (generators, batteries, loads), hard reactive power limits
- **Tap changers** at the substation transformers

# An "autonomous" feedback control design problem



- **Steady state specifications:** solution of a constrained optimization problem
- **Schedule:** known parameter
- **Disturbance:** unknown parameters

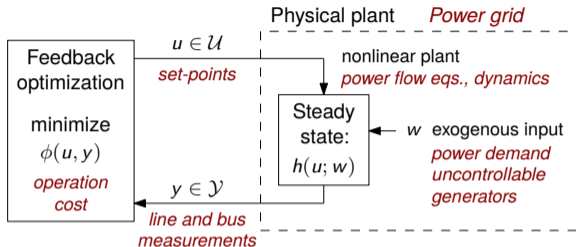
→ **Feedback optimization**

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# Feedback form of the OPF problem



## Steady-state specifications

$$\begin{aligned} & \text{minimize}_{u, y} && \phi(u, y) \\ & \text{subject to} && y \in \mathcal{Y} \\ & && u \in \mathcal{U} \\ & && y = h(u; w) \end{aligned}$$

## Optimization perspective

Analysis and design of algorithms with the tools of dynamical systems  
**but we implement them via the physics**

## Control perspective

Feedback systems interpreted as solvers of a specific optimization problem **but we require general objective + constraints**

**Related:** Self-optimizing control, economic MPC, real-time iteration, modifier adaptation, extremum seeking,...

Preprint "Optimization Algorithms as Robust Feedback Controllers" (2021) [↗](#)

# Steady-state map $y = h(u; w)$

**Chart** for the  $2n$ -dimensional manifold of **power flow equations**:  
invertible map between  $\mathbb{R}^{2n}$  and a open subset of  $\mathcal{M}$

## Implicit function theorem

If a manifold is defined as

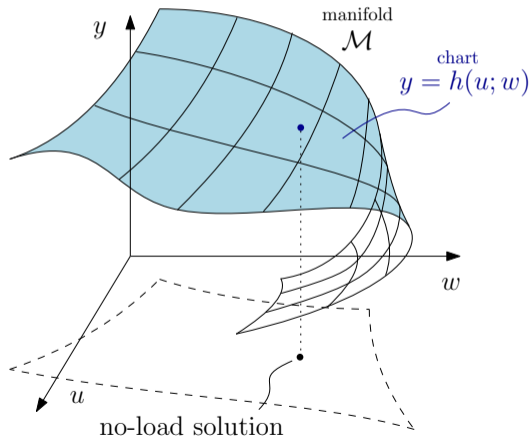
$$\mathcal{M} = \{(u, w, y) \mid F(u, w, y) = 0\}$$

then there exists a continuously differentiable function  $y = h(u, w)$  such that

$$F(u, w, h(u, w)) = 0$$

in the open subset where

$$\nabla_y F(u, w, y) \text{ is invertible}$$



# Input-output sensitivities $\nabla_{u,w}h(u, w)$

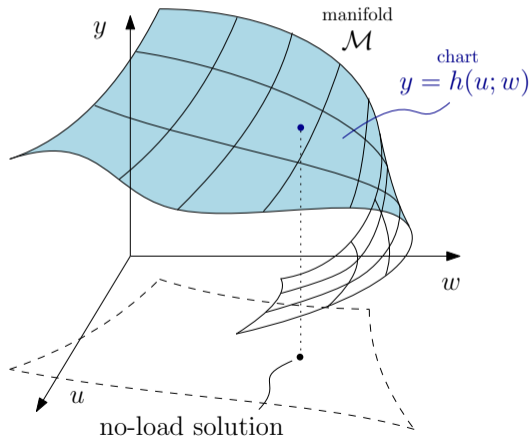
$$\nabla_{u,w}h(u, w) = -(\nabla_y F(u, w, y))^{-1} \nabla_{u,w} F(u, w, y)$$

$\nabla_y F(u, w, y)$  is known as the power flow Jacobian and connected to

- power flow solvability
- voltage collapse

## High-voltage PFM

Largest connected component of  $\mathcal{M}$  that contains the **no-load solution** and where  $\nabla_y F(u, w, y)$  is invertible



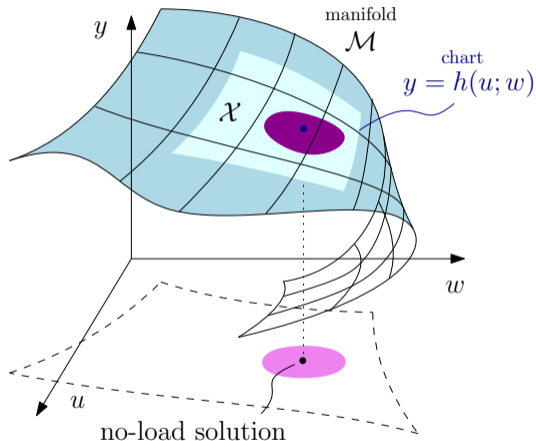
# High-voltage PFM

- The high-voltage PFM  $\mathcal{M}_{\text{high}}$  can rarely be derived in closed form
  - 2-bus example and little else
- **Inner approximations** are available, but they are usually **conservative**

## Running assumption

The operational constraints guarantee that the state of the grid belongs to the high-voltage region

$$(\mathcal{U} \times \mathcal{Y} \times \mathcal{W}) \cap \mathcal{M} \subset \mathcal{M}_{\text{high}}$$



# Design of feedback optimizers

Borrow ideas from **iterative optimization algorithms** for **non-convex optimization** and interpret these algorithms as dynamical systems

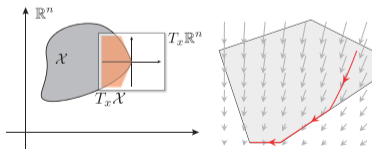
- Gradient Flows  
[Brockett, 1991], [Bloch et al., 1992], [Helmke & Moore, 1994], ...
- Interior-point methods  
[Karmarkar, 1984], [Khachian, 1979], [Faybusovich, 1992], ...
- Acceleration & Momentum methods  
[Su et al., 2014], [Wibisono et al., 2016], [Krichene et al., 2015], [Wilson et al., 2016], [Lessard et al., 2016], ...
- Saddle-Point Flows  
[Arrow et al., 1958], [Kose, 1956], [Feijer & Paganini, 2010], [Cherukuri et al., 2017], [Holding & Lestas, 2014], [Cortés & Niederländer, 2018], [Qu & Li, 2018], ...

**Claim:** In continuous-time, most algorithms reduce to either (projected) **gradient flows** (w/o momentum) or (projected) **saddle-point** flows.

# Examples of feedback optimization design

$$\begin{aligned} & \text{minimize}_{u,y} && \phi(u, y) \\ & \text{subject to} && y \in \mathcal{Y} \quad \text{output constraints} \\ & && u \in \mathcal{U} \quad \text{input saturation} \\ & && y = h(u; w) \quad \text{power flow equations} \end{aligned}$$

$$\begin{aligned} & \text{minimize}_u && \phi(u, y) + p(y) \\ & \text{subject to} && u \in \mathcal{U} \\ & && y = h(u; w) \end{aligned}$$



$\mathcal{Y} \rightarrow$  Penalty function (Hauswirth 2017, Tang 2017, Mazzi 2018, ...)

Gradient descent flow  $\rightarrow$  proportional-like feedback law

$$\dot{u} = \Pi_{\mathcal{U}} \left[ -\nabla_u \phi(u, y) - \underbrace{\nabla h(u; w)' \nabla_y \phi(u, y)}_{\text{model}} - \underbrace{\nabla h(u; w)' \nabla p(y)}_{\text{model}} \right]$$

$\rightarrow$  **arbitrarily small output constraint violation**

# Examples of feedback optimization design

$$\begin{aligned} & \text{minimize}_{u,y} && \phi(u, y) \\ & \text{subject to} && y \in \mathcal{Y} \quad \text{output constraints} \\ & && u \in \mathcal{U} \quad \text{input saturation} \\ & && y = h(u; w) \quad \text{power flow equations} \end{aligned}$$

Output constraint  
representation

$$\mathcal{Y} := \{y \mid g(y) \leq 0\}$$

Lagrangian

$$\mathcal{L}(u, y, \lambda) = \phi(u, y) + \lambda' g(y)$$

**Saddle flow** (Bolognani 2015, Dall'Anese 2018, Bernstein 2019, Colombino 2020, ...)

Primal descent / dual ascent  $\rightarrow$  proportional-integral feedback law

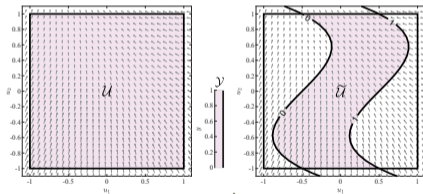
$$\begin{cases} \dot{u} = \Pi_{\mathcal{U}} \left[ -\nabla_u \phi(u, y) - \underbrace{\nabla h(u; w)'}_{\text{model}} \nabla_y \phi(u, y) - \underbrace{\nabla h(u; w)'}_{\text{model}} \nabla g(y)' \lambda \right] \\ \dot{\lambda} = \Pi_{\geq 0} [g(y)] \end{cases}$$

$\rightarrow$  **asymptotic (exact) constraint satisfaction**

# Examples of feedback optimization design

$$\begin{aligned} & \text{minimize}_{u,y} && \phi(u, y) \\ & \text{subject to} && y \in \mathcal{Y} \quad \text{output constraints} \\ & && u \in \mathcal{U} \quad \text{input saturation} \\ & && y = h(u; w) \quad \text{power flow equations} \end{aligned}$$

$$\tilde{\mathcal{U}} = \mathcal{U} \cap h^{-1}(\mathcal{Y})$$



Projected gradient descent (Hauswirth 2016, Häberle 2020, ...)

Projection on the input and output constraints

$$\dot{u} = \Pi_{\tilde{\mathcal{U}}} \left[ -\nabla_u \phi(u, y) - \underbrace{\nabla h(u; w)'}_{\text{model}} \nabla_y \phi(u, y) \right]$$

→ any-time constraint satisfaction



# Projected gradient flow via repeated Quadratic Programming

Continuous-time flow

$$\dot{u} = \Pi_{\mathcal{U}} \left[ -\nabla_u \phi(u, y) - \underbrace{\nabla h(u; w)'}_{\text{model}} \nabla_y \phi(u, y) \right]$$

Assumption

$$\mathcal{U} := \{u \in \mathbb{R}^p \mid Au \leq b\}$$

$$\mathcal{Y} := \{y \in \mathbb{R}^n \mid Cy \leq d\}$$

Discrete-time approximation

$$u^+ = u + \delta u \quad \text{where} \quad \delta u := \arg \min_v \quad \|v - (-\nabla_u \phi(u, y) - \nabla h(u; w)' \nabla_y \phi(u, y))\|^2$$

subject to  $A(u + v) \leq b$   
 $C(y + \nabla h(u; w)' v) \leq d,$

1st order approx of  $h^{-1}(\mathcal{Y})$  centered at the measurement  $y$

**Theorem:** V. Häberle et al., “Non-convex Feedback Optimization with Input and Output Constraints,” 2020 [↗](#)

LICQ + Lipschitz + differentiability + small  $\rightarrow$  global convergence to the set of local minima

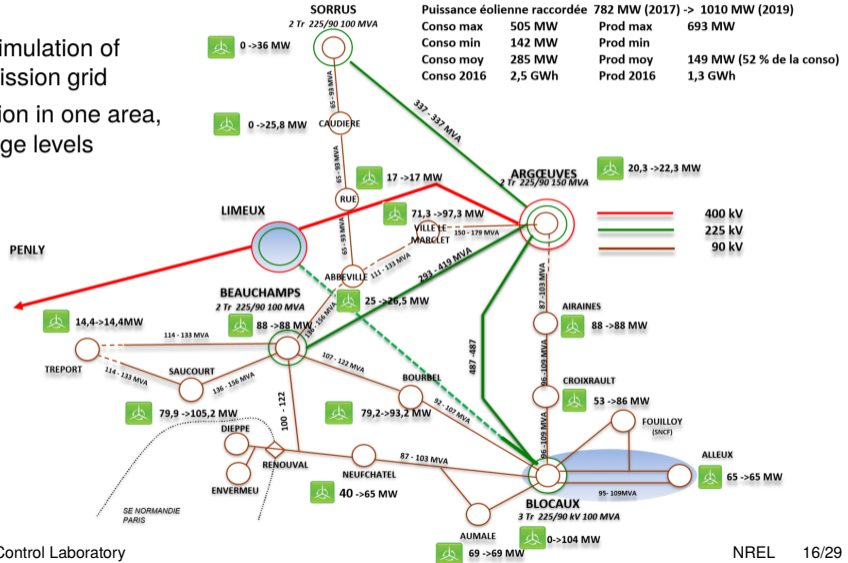
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# Benchmark (soon to become public)

Quasi-steady state simulation of entire French transmission grid

**Goal:** avoid congestion in one area, across different voltage levels



# Problem specifications

## Inputs

### Uncontrollable

- Distributed wind generation (historical worst-case ramp)

### Controllable

- Transformers tap-changer position
- Wind generators reactive power injection
- Active power curtailment

**Scenario:** 225kV-90kV transformer offline

Other scenarios: no tap changers, tighter constraints, higher generation, ...

## Output

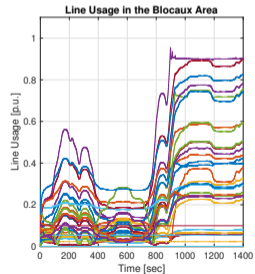
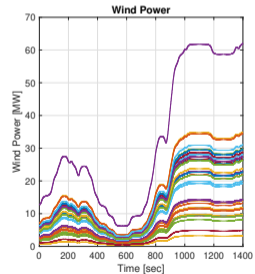
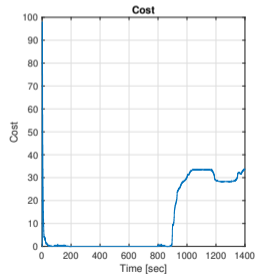
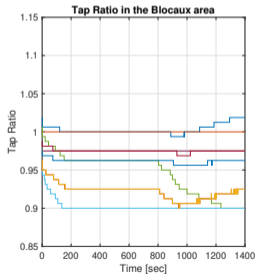
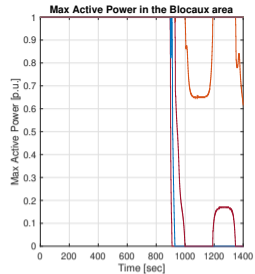
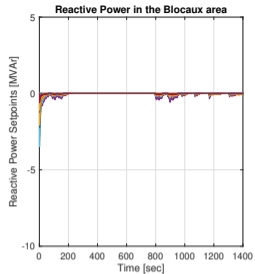
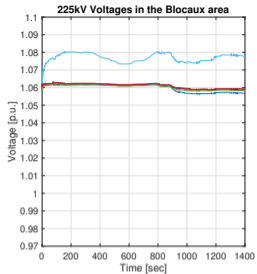
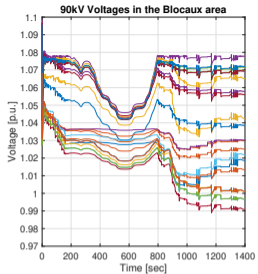
- real-time area state estimation

## Constraints

- bus voltage limits
- line current limits
- generator limits
- tap changes

## Cost

- \$\$\$ Active power curtailment
- \$ Power losses



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# How about emergencies?



→ Overview of the events and causes of the 2003 Italian blackout ↗

03:01	Trip of the 380 kV line Mettlen Lavorgo (CH). Attempts to reclose the line automatically until 3.03:50. Manual re-closure fails at 3:03:50.
03:02-03:08	Attempts to reclose the Mettlen-Lavorgo line. Information exchanges between ETRANS and ATEL and EGL dispatchers.
03:10	ETTRANS, by phone, requests a reduction of 300 MW in Italian imports to scheduled values.
03:18-03:22	Exchange of information ETRANS - ATEL -EGL; changes in topology of the Swiss system.
03:21	Italian imports are reduced to 6400 MW
03:25	Trip of the Sils-Soazza 380 kV line (CH)
03:25	Trip of the Airolo Mettlen 220 kV line (CH)

→ 1.2 billion EUR

# How about emergencies?



→ Overview of the events and causes of the 2003 Italian blackout [↗](#)

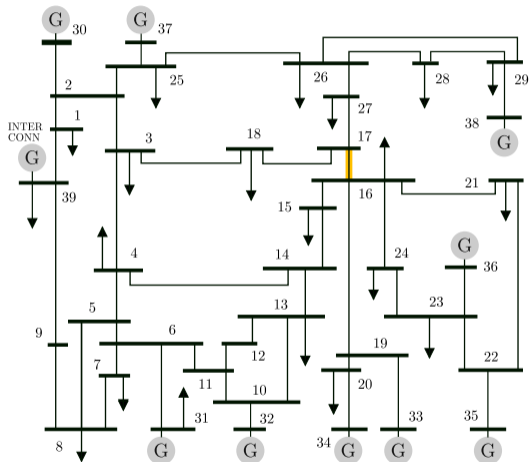
**Can we frame emergency grid operation as a feedback optimization problem?**

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# Numerical experiment IEEE 39 test grid



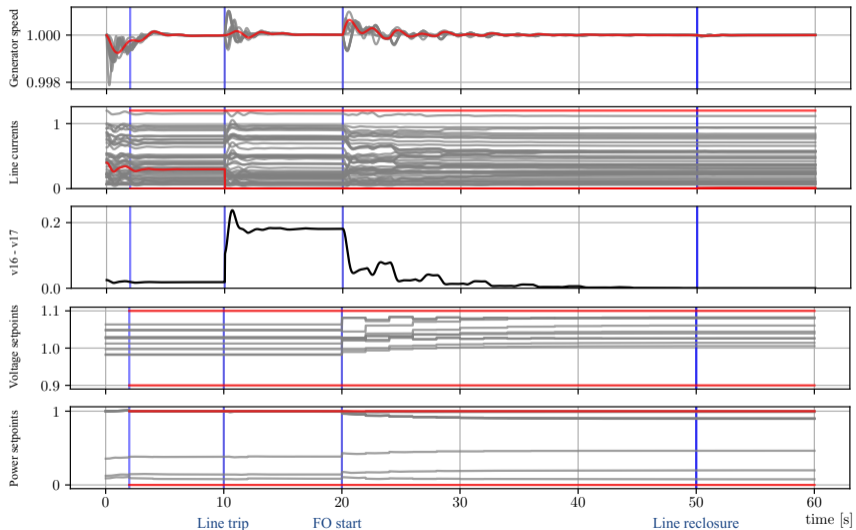
- **Fault:** Line trip of line 16-17
- **Goal:** Reclosure before cascading failure
- Line reclosure requires **small voltage difference** at the breaker

## FO formalization

$$\begin{aligned}
 & \min_{P_G, V_G} \|v_{16} - v_{17}\|^2 \\
 & \text{subject to} \quad 0 \leq P_{G_i} \leq P_{G_i}^{\max} \\
 & \quad V_{G_i}^{\min} \leq V_{G_i} \leq V_{G_i}^{\max} \\
 & \quad v^{\min} \leq v_i \leq v^{\max} \\
 & \quad 0 \leq f_{ij} \leq f_{ij}^{\max}
 \end{aligned}$$

# Full dynamic simulation

DynPSSimPy [↗](#) (by Gianni Hotz)



# Feedback optimization for emergencies

The problem is... it shouldn't work!


## Grid dynamics not at steady-state between set-point updates

- Design/certify stable interconnections (LTI systems)  
Lawrence, Simpson-Porco, Mallada, "*Linear-convex optimal steady-state control*", 2020 [↗](#)  
Colombino, Dall'Anese, Bernstein, "*Online opt. as a feedback controller: stability and tracking*," 2018 [↗](#)  
Bianchin et al. "*Time-varying optimization of LTI systems via projected primal-dual gradient flows*," 2021 [↗](#)
- **Quantify sufficient time-scale separation (nonlinear grid dynamics)**

## Wrong (pre-fault) model during contingency

- Rely on the inherent robustness of feedback optimization (performance tradeoff)  
Colombino, Simpson-Porco, Bernstein, "*Towards robustness guarantees for feedback-based opt.*," 2019 [↗](#)  
L. Ortmann et al., "*Experimental validation of feedback optimization in power distribution grids*," 2020 [↗](#)
- **Online sensitivity estimation**

# Time-scale separation analysis

A. Hauswirth, S. Bolognani, G. Hug, F. Dörfler, IEEE TAC, 2021 

## Optimization Dynamics

The cost function  $\phi(u, x)$  has

- compact level sets
- **$L$ -Lipschitz** gradient.

Unconstrained gradient descent

$$\dot{u} = - \left( -\nabla_u \phi(u, x) - \nabla h(u; w)' \nabla_y \phi(u, x) \right)$$

## Plant Dynamics

Exponentially stable system

$$\dot{x} = f(x, u; w) \quad (\text{steady state } x = h(u; w))$$

with Lyapunov function  $W(u, x)$  such that

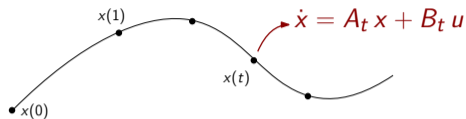
$$\begin{aligned} \dot{W}(u, x) &\leq - \|x - h(u; w)\|^2 \\ \|\nabla_u W(u, x)\| &\leq \zeta \|x - h(u; w)\|. \end{aligned}$$

Then, all trajectories converge to the set of KKT points whenever

$$< \frac{1}{\zeta L}.$$

(Similar results for projected gradient, saddle flow, Newton flow. **Not** for subgradient, accelerated gradient.)

# Numerical evaluation at multiple linearization points



- solve **Lyapunov eq.**  $A_t^\top P_t + P_t A_t \preceq I$
- **Lyapunov fcn.**  $W(u, x) = \|x - h(u)\|_{P_t}^2$
- $\dot{W}(u, x) \leq -\|x - h(u)\|^2$  ( $\rightarrow = 1$ )
- **Linear steady state**  $h(u) = H u$
- $\zeta = \|P_t H\|$

$$\rightarrow \leq \frac{1}{L \|P_t H\|}$$

## Plant Dynamics

### Exponentially stable system

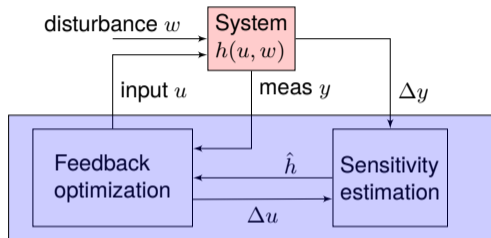
$$\dot{x} = f(x, u; w) \quad (\text{steady state } x = h(u; w))$$

with Lyapunov function  $W(u, x)$  such that

$$\begin{aligned} \dot{W}(u, x) &\leq -\|x - h(u; w)\|^2 \\ \|\nabla_u W(u, x)\| &\leq \zeta \|x - h(u; w)\|. \end{aligned}$$

- + bound on  $\zeta$  relatively uniform in  $x$
- very conservative certificate

# Online sensitivity estimation



Best online estimate

$$\hat{h}_{t+1} = \arg \min_{\hat{h}} \hat{h} - \hat{h}_t \frac{\Sigma_t^{-1}}{\|\Delta u_t\|_2^2} + \Delta y_t - U_{\Delta,t} \hat{h} \frac{\Sigma_{m,t}^{-1}}{\|\Delta u_t\|_2^2}$$

→ Kalman-like update

$$\hat{h}_{t+1} = \hat{h}_t + K_t (\Delta y_t - U_{\Delta,t} \hat{h}_t)$$

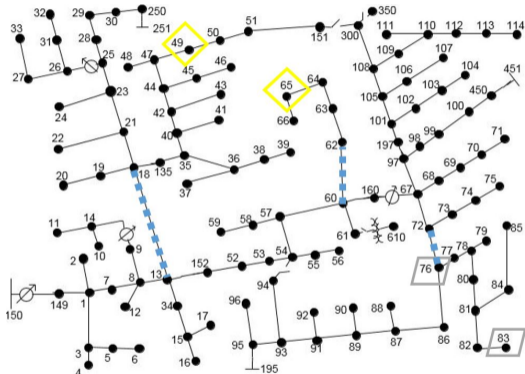
$$\Sigma_{t+1} = (1 - K_t U_{\Delta,t}) \Sigma_t + \Sigma_{p,t} \|\Delta u_t\|_2^2,$$

## Proposition

**Strong-monotonicity** of the optimization flow + **persistently exciting** input  $u_t \Rightarrow$

$$\lim_{t \rightarrow \infty} \mathbb{E}[\|h_t - \hat{h}_t\|_2^2] \rightarrow 0 \quad \lim_{t \rightarrow \infty} \mathbb{E}[\|h_t - \hat{h}_t\|_2^2] \rightarrow C_h < \infty \quad \lim_{t \rightarrow \infty} \mathbb{E}[\|u_t - u^*(d_t)\|_2^2] \rightarrow C_u \leq \dots$$

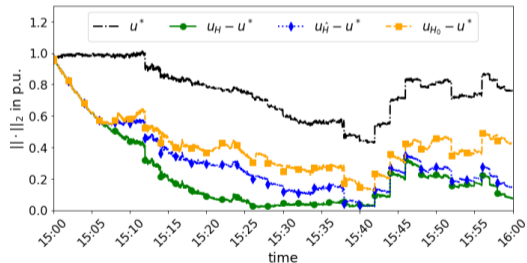
# Numerical experiment



IEEE 123 test case with

- modified line impedance
- nonlinear regime

→ **time-varying steady-state sensitivity**



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# Highlights

- **Real-time operation of power systems** can be automated via **feedback optimization**
  - UNICORN numerical testbed will be published soon
- Feedback optimization design taps into **iterative nonlinear optimization algorithms**
- **Numerical experiments** show that feedback optimization can produce complex multi-input/multi-objective responses to contingencies
- **Open problem 1:** tighter stability certificates for the interconnected systems
  - design guidelines
  - uniform in the system working point
- **Open problem 2:** online sensitivity estimate
  - model-free design
  - robustness to unforeseen system changes



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