Online Optimization of Switched LTI Systems

Emiliano Dall'Anese

University of Colorado
Boulder

NREL Workshop on Autonomous Energy Systems | August 20, 2020

Acknowledgments

Gianluca Bianchin

Jorge I. Poveda

Reference: G. Bianchin, J. I. Poveda, and E. Dall'Anese, "Online Optimization of Switched LTI Systems Using Continuous-Time and Hybrid Accelerated Gradient Flows," 2020. [Online] https://arxiv.org/abs/2008.03903.

Supported by: NREL via UGA-0-41026-136, NSF CAREER award #1941896, NSF CNS #1947613

Problem Formulation

Switched LTI plant

$$\dot{x} = A_\sigma x + B_\sigma u + E_\sigma w$$
$$y = Cx + Dw$$

$w \rightarrow$ Unknown disturbance
$u \rightarrow$ Control input
$y \rightarrow$ Output
$\sigma(t) \rightarrow$ Switching signal

Objective: output regulation

$$\min_{u,x} \quad g(u) + h(y)$$
$$\text{s.t.} \quad 0 = A_\sigma x + B_\sigma u + E_\sigma w$$
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Regulate the plant to the equilibrium points that minimize a given cost

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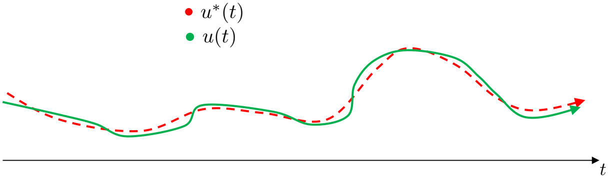
$$\begin{aligned} \min_{u,x} \quad & g(u, t) + h(y, t) \\ \text{s.t.} \quad & 0 = A_\sigma x + B_\sigma u + E_\sigma w(t) \\ & y = Cx + Dw \end{aligned}$$

Output tracks optimal solution trajectory implicitly defined by a time-varying problem



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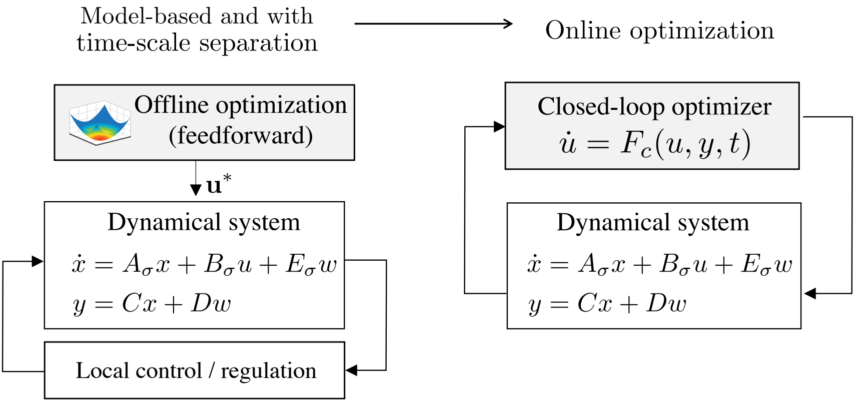
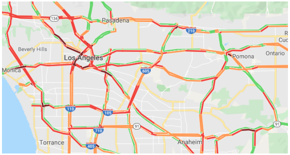
Key quantity in this talk: $\dot{w} \to$ time-variability of an optimal solution
I.e., $\dot{w} \leftrightarrow \dot{u}^*$

Algorithmic design

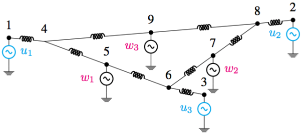


Motivating Examples



$\min$ Total Travel (Cost, Time)
s.t. Network is at equilibrium
(Using real-time measurem.)

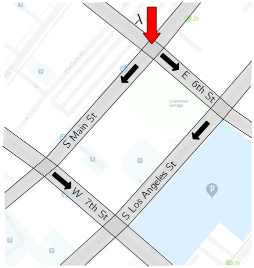
[Bianchin-Pasqualetti'20], [Grandinetti et al '18]



$\min$ Total generation cost
s.t. Swing equation
(Using real-time measurem.)

[Menta et al'19], [Colombino et al'20]

Suboptimal controllers (i.e., route selection) $\rightarrow$ instabilities

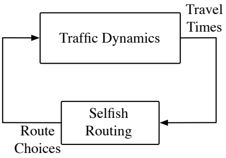


Two parallel roads:

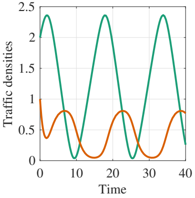
$$\dot{x}_1 = -f(x_1) + r\lambda$$
$$\dot{x}_2 = -f(x_2) + (1-r)\lambda$$

Drivers select the fastest route:

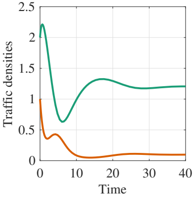
$$\dot{r} = r(1-r)(x_2 - x_1)$$



"free" selfish routing

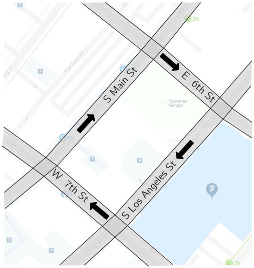


With controller



Motivating Examples From Traffic Control (2)

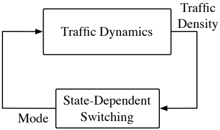
Uncontrolled traffic → instabilities



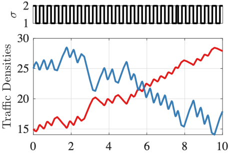
A more accurate model: CTM

$$\dot{x}_1 = -\min\{d_1 x_1, s_2 x_2\} + r_{21}\min\{d_2 x_2, s_1 x_1\} + w_1$$
$$\dot{x}_2 = -\min\{d_2 x_2, s_1 x_1\} + r_{12}\min\{d_1 x_1, s_2 x_2\} + w_2$$

Roads have two modes: free-flow and congested



Uncontrolled system



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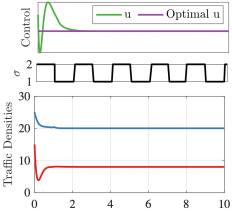

Related Work (a short list)

- Related to feedback-based optimization (with plant ∞ fast)

  [Jokic et al'09], [Bolognani et al'13], [Bernstein et al'13], [Dall'Anese-Simonetto'16], [Bernstein et al'19], [Colombino et al'19], ...

- [Menta et al'19]: LTI, static optimization, stability of strict local optima

- [Hauswirth et al'20] extension to nonlinear dynamical systems

- [Lawrence et al'18] Joint stabilization and regulation, static, LTI

- [Colombino et al'18] LTI, time-varying, strongly convex, exponential ISS

- [Zheng et al'19] Feedback-linearizable systems, time-varying

Today: switched LTI, time-varying, exponential ISS, gradient flow

In the paper: Hybrid Nesterov, practical stability, other results ...

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Design of Feedback Controllers

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Recall: gradient flow (if plant is ∞ fast and w is known)

$$\begin{aligned} y &= -CA^{-1}Bu + (D - CA^{-1}E)w \\ \dot{u} &= -\nabla g(u) - G^\top \nabla h(Gu + Hw) \end{aligned}$$

"solves" the optimization

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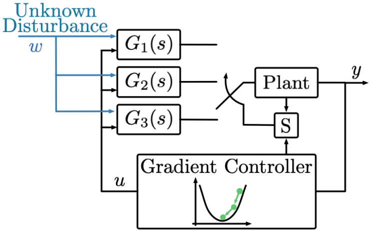
Proposed online gradient-based controller:



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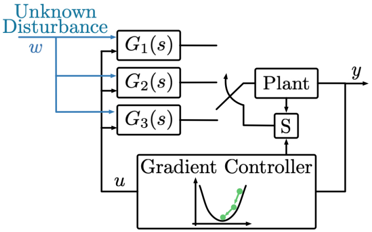
Framework: Hybrid Dynamical Systems



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Hybrid dynamical system (HDS):

Flow: $\dot{z} \in F(z, \dot{w})$ Jumps: $z^+ \in G(z, \dot{w})$

HDS for interconnection between gradient flow and plant:

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$$\dot{\tau} \in [0, 1/\tau_d]$$
$\tau_d \rightarrow$ average "dwell time"

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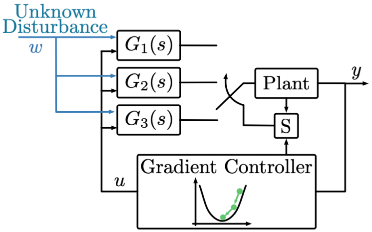

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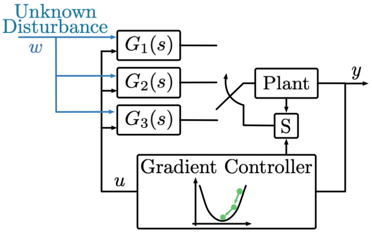

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ISS for Hybrid Dynamical Systems

HSD with flows $\dot{z} \in F(x, \dot{w})$ and jumps: $\dot{z}^+ \in G(x, \dot{w})$

Definition [Nešić et al '13]. The compact set $\mathcal{A}$ is ISS if:

$$\|z(t,j)\|_{\mathcal{A}} \leq \beta(\|z(0,0)\|_{\mathcal{A}}, t, j) + \gamma(\sup_{\tau \geq 0} \|\dot{w}(\tau)\|)$$

where $\beta \in \mathcal{KLL}$ and $\gamma \in \mathcal{K}_\infty$.

Definition. The set $\mathcal{A}$ is exponentially ISS (E-ISS) if:

$$\|z(t,j)\|_{\mathcal{A}} \leq a_0(e^{-\frac{1}{2}(b_0 t + c_0 j)}\|z(0,0)\|_{\mathcal{A}} + d_0 \sup_{\tau \geq 0} \|\dot{w}(\tau)\|)$$

If the set of oprimizers are E-ISS for the feedback interconnection, then the controller achieves tracking and stability

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Technical Assumptions: Switched LTI system

As. Individual modes are exponentially stable:

$$A_\sigma^\top P_\sigma + P_\sigma A_\sigma \preceq -Q_\sigma, \quad P_\sigma \succ 0, Q_\sigma \succ 0$$

As. Modes have common equilibrium points:

$$\exists\, u, w \,|\, 0 = A_\sigma x + B_\sigma u + E_\sigma w \text{ for all } \sigma$$

Note: We focus switching signals for which the number of discontinuities in every open interval $(s,t) \subset \mathbb{R}_+$ satisfies [Hespanha-Morse '99] :

$$N(t,s) \leq N_0 + \tfrac{t-s}{\tau_d}$$

Technical Assumptions: Cost

As. The cost functions have a Lipschitz-continuous gradient:

$$\|\nabla h(u) - \nabla h(u')\| \leq \ell_u \|u - u'\| \qquad \|\nabla g(y) - \nabla g(y')\| \leq \ell_y \|y - y'\|$$

As. Cost satisfies Polyak-Łojasiewicz (PL) inequality:

$$\frac{1}{2}\|\nabla f(u)\|^2 \geq \mu(f(u) - f(u^*))$$

A strongly convex function satisfies the PL inequality, but the inverse implication does not hold. PL inequality implies invexity [Karimi et al'16]

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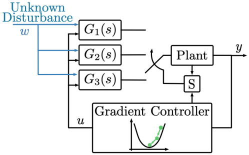
Tracking: Single Mode



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Define: $z = [u, x]$, $z^* = [u^*, x^*]$

Proposition. Suppose that the plant has a single mode. If

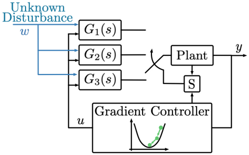
$$\varepsilon_\sigma < \frac{\underline{\lambda}(Q_\sigma)}{4\ell_y \|C\| \|G\| \|P_\sigma A_\sigma^{-1} B_\sigma\|}$$

then the set $\{z - z^*\}$ is E-ISS with rate $b_0 = \mu^2/\ell$, $c_0 = 0$.

Details: $a_0 = \sqrt{\bar{a}_\sigma / \underline{a}_\sigma}$, $\bar{a}_\sigma = \max\left\{(1-\theta_\sigma)\frac{\ell}{2}, \theta_\sigma \bar{\lambda}(P_\sigma)\right\}$, $\underline{a}_\sigma = \min\left\{(1-\theta_\sigma)\frac{\mu}{2}, \theta_\sigma \underline{\lambda}(P_\sigma)\right\}$

$d_0 = \frac{\sqrt{2}\max_\sigma \|r_\sigma\|}{\tilde{d}\min\{1,\mu^2\}}$, $r_\sigma^\top = [2\theta_\sigma \|P_\sigma A_\sigma^{-1} E_\sigma\|, (1-\theta_\sigma)\ell_y \|H\| \|G\|]$

Tracking: Single Mode



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Theorem. Consider the interconnection above with state $(\tilde{z}, \tau, \sigma)$, where $\tilde{z} = z - z^*$ is the tracking error. If

$$\varepsilon_\sigma < \frac{\underline{\lambda}(Q_\sigma)}{4\ell_y \|C\| \|G\| \|P_\sigma A_\sigma^{-1} B_\sigma\|} \quad \text{and} \quad \tau_d > \frac{\ell}{2\mu^2} \ln\left(\frac{\bar{a}}{\underline{a}}\right), \; N_0 \in \mathbb{Z}_{>0}$$

then the set $\mathcal{A} = \{0\} \times \mathcal{S} \times \mathcal{T}_C$, with $\mathcal{T}_C := [0, N_0] \times \mathcal{S}$, is E-ISS with parameters $b_0 = \mu^2/\ell - \frac{\varrho}{\tau_d}$, $c_0 = \varrho - \ln\left(\frac{\bar{a}}{\underline{a}}\right)$.

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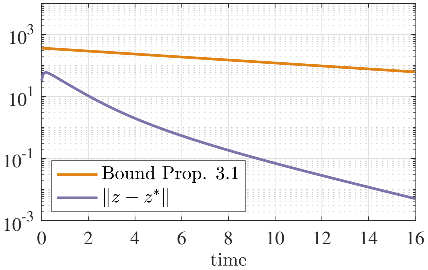
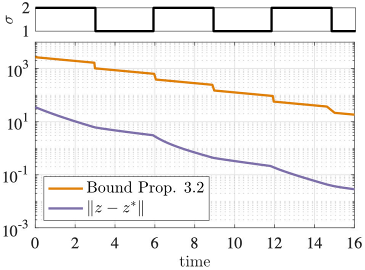
$d_0 = \frac{\sqrt{2} \max_\sigma \|r_\sigma\|}{\tilde{d} \min\{1, \mu^2\}}$

Constant Disturbance: Illustrations

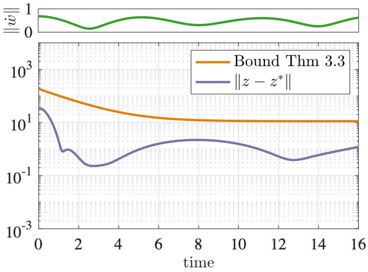
$$\|z(t,j)\|_{\mathcal{A}} \quad \leq \quad a_0 e^{-\frac{1}{2}(b_0 t + c_0 j)} \|z(0,0)\|_{\mathcal{A}}$$

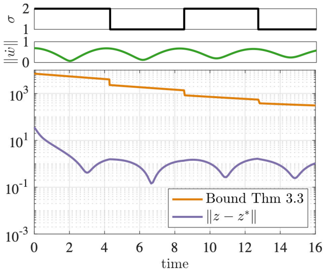




Tracking: Illustrations

$$\|z(t,j)\|_{\mathcal{A}} \;\leq\; a_0 e^{-\frac{1}{2}(b_0 t + c_0 j)} \|z(0,0)\|_{\mathcal{A}} + d_0 \sup_{\tau \geq 0} \|\dot{w}(\tau)\|)$$





Teaser: Hybrid Restarted Nesterov

Hybrid feedback controller inspired by Nesterov's accelerated gradient

$$\varepsilon_\sigma \dot{x} = A_\sigma x + B_\sigma u_1 + E_\sigma w$$

$$\dot{u}_1 = \frac{\rho}{u_3}(u_2 - u_1)$$

$$\dot{u}_2 = -\frac{\kappa u_3}{\rho}\left(\nabla h(u_1) + G^\top \nabla g(y)\right) \qquad u_2^+ = r_0 u_1 + (1 - r_0) u_2$$

$$\dot{u}_3 = 1 \qquad u_3^+ = T$$

1) Resolves the lack of robustness of accelerated methods when online and in closed loop with dynamical system [Hauswirth et al '20]

2) Outperforms the gradient-flow controller for strongly convex costs

See our paper!

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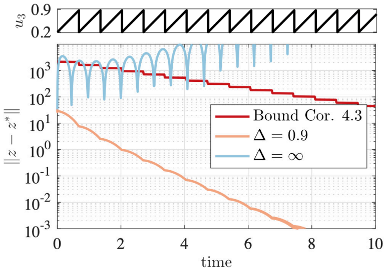

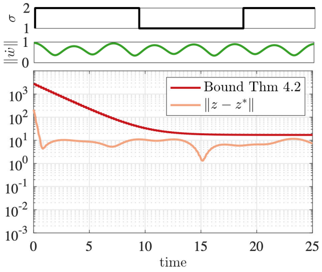

Conclusions

- Work at the intersection of online optimization, control, and HDS
- E-ISS for switched LTI and costs that satisfy the PL inequality
- Under strong convexity, hybrid restarted Nesterov method
- Next steps:

  - Constrained problems
  - Robustness to inexact gradient information
  - Connections with MPC (with J. Cortes)

Thanks!