Decentralized Integration of Solar and Storage into Wholesale Energy Markets via Mean-field Games

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September 3, 2024

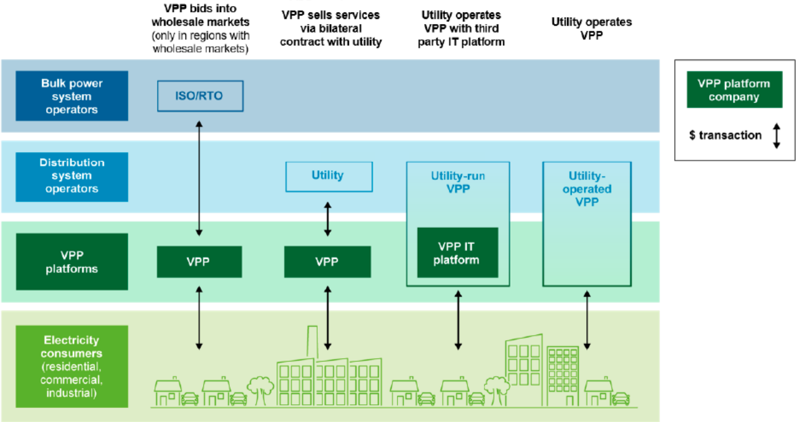
Outline

- General overview of aggregators/virtual power plants/DSOs
- Model Building blocks
  - (i) a wholesale market (LMPs);
  - (ii) individual prosumers' MDP problem
- Mean-field game of multiple prosumers
  - (i) existence of a mean-field equilibrium (MFE);
  - (ii) a heuristic algorithm
- Numerical results

Part I – Conceptual Framework

Virtual Power Plant (VPP) Business Models

"VPPs are aggregations of DERs that can balance electrical loads and provide utility-scale and utility-grade grid services like a traditional power plant."



Source: DOE. Pathways to Commercial Liftoff: Virtual Power Plants. September, 2023

Companies Provide VPP Services

	Data/IT Platform of VPP Operator	Market interface	DER platforms
Examples (not exhaustive)	AutoGrid CPower Nest OhmConnect swell TESLA sunrun voltus	AutoGrid CPower OhmConnect RECURVE swell voltus	AutoGrid CPower Nest swell sunrun TESLA voltus

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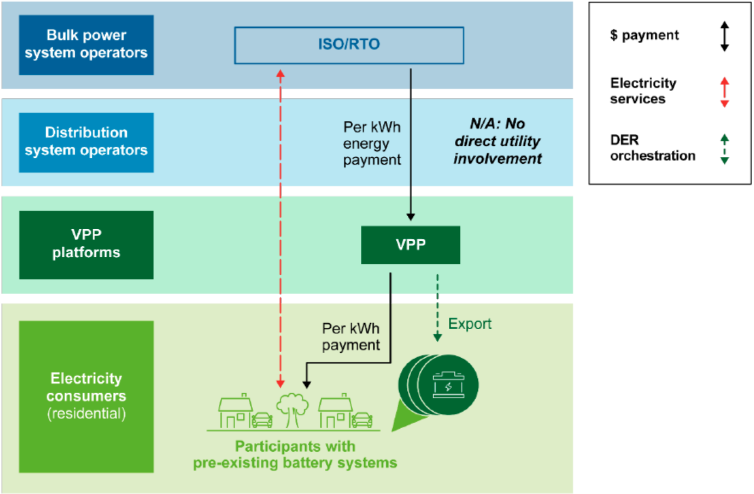
Our Focus: VPPs into Wholesale Markets



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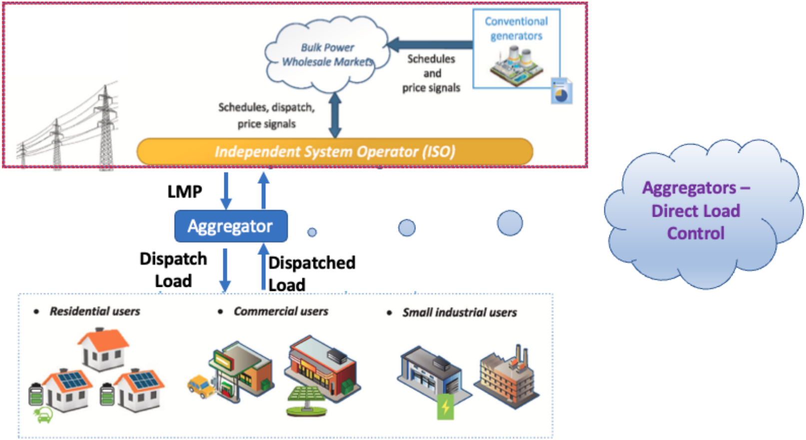
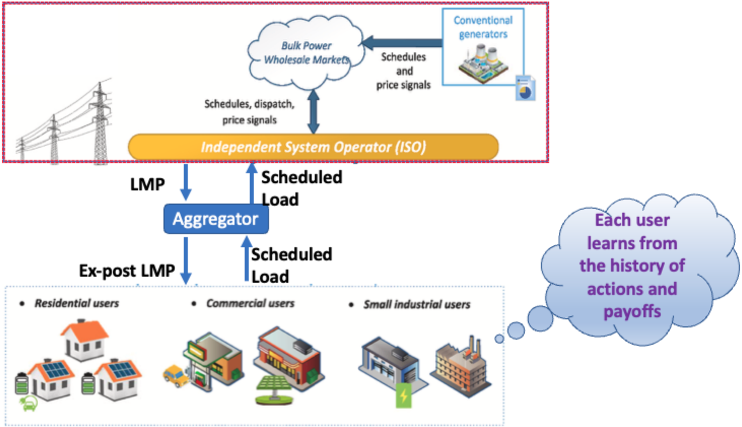
VPP to Wholesale Approach 1: Direct Load Control



VPP to Wholesale Approach 2: Decentralized Aggregation



Key Challenges of the Decentralized Approach

- Repeated interactions
- Lacks of expertise/resources/time to participate
- The problem with energy storage is hard
- Distribution system feasibility (outside the scope of this work)

Part II – Models

- II.1 Wholesale market model
- II.2 Prosumer's MDP

The Wholesale Market Model and LMPs

Economic Dispatch Problem

$$\min_{\vec{g}_t} \sum_{n=1}^{N} C_n(g_t^n) \; - \text{ minimize total generation cost}$$

$$\text{s.t.} \sum_{n=1}^{N} g_t^n \geq \sum_{n=1}^{N} \sum_{i=1}^{I_n} b_{i,t}^n \; - \text{ supply/demand balancing (dual : } \lambda)$$

$$-\widehat{\mathrm{F}}_l \leq \sum_{n=1}^{N} \mathrm{PTDF}_{l,n}(g_t^n - \sum_{i=1}^{I_n} b_{i,t}^n) \leq \widehat{\mathrm{F}}_l, \; \forall l \in \{1, ..., L\}$$

$$- \text{ transmission constraints (dual : } \overline{\mu}_l, \; \underline{\mu}_l)$$

$$0 \leq g_t^n \leq \widehat{\mathrm{G}}_n, \quad \forall n \in \{1, ..., N\}, \; - \text{ generation capacity constraints}$$

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LMPs

$$\mathrm{LMP}_t^n := P^n(B_t^1, \ldots, B_t^N) = \frac{\partial \mathcal{L}}{\partial B_t^n} = \lambda - \sum_{l=1}^{L} \mathrm{PTDF}_{l,n}(\overline{\mu}_l - \underline{\mu}_l).$$

Lipschitz Continuity of LMPs w.r.t. Demand

Assumption: LICQ

represent the vector $(B_t^1, \dots, B_t^N)$, with $B_t^n = \sum_{i=1}^{I_n} b_{i,t}^n$. Let $X(\mathbf{B}_t)$ denote the feasible region of the Economic Dispatch problem. We assume that for all t and for all $\mathbf{B}_t \in \mathcal{F}_B$, the linear independence constraint qualification (LICQ) holds at all points in $X(\mathbf{B}_t)$

Lipschitz Continuity of LMPs

Proposition. Assume that the generation cost function $C_n(\cdot)$ is a strongly convex quadratic function in the form of $C_n(g) = \frac{1}{2}\alpha_n g^2 + \beta_n g + \gamma_n$, with $\alpha_n > 0$ for all $n = 1, \dots, N$. Under the LICPQ Assumption, with $\mathbf{B}_t \in \mathcal{F}_B$, the LMP at each node $n = 1, \dots, N$, $P^n(\mathbf{B}_t)$, is a single-valued function and Lipschitz continuous with respect to $\mathbf{B}_t$.

The MDP of Prosumer i of Type θ

- Action: $a_{i,t} \in [-100\%, 100\%]$ charging ($a_{i,t} > 0$)/discharging ($a_{i,t} < 0$)

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$$
\begin{aligned}
e_{i,t} &:= E(e_{i,t-1}, a_{i,t-1}) \\
&= \max\{\min\{e_{i,t-1} + a_{i,t-1}, 1\}, 0\}, \ \ t = 1, 2, \ldots,
\end{aligned}
$$

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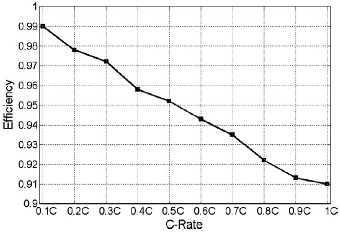
- Chargin/discharging Efficiency:



Prosumer's MDP (cont.)

- Chargin/discharging Efficiency (cont):

$$\eta(a_{i,t}) = \begin{cases} \alpha_0 + \alpha_d \cdot a_{i,t}, & \text{if } a_{i,t} < 0, \\ \alpha_0 - \alpha_c \cdot a_{i,t}, & \text{if } a_{i,t} \geq 0, \end{cases}$$

- Prosumer's bids:

$$b^{\theta}_{i,t}(e_{i,t}, a_{i,t}, q^{\theta}_{i,t}) =$$

$$\begin{cases} Q^{\theta}_{i,t} + \eta(a_{i,t})\overline{e}^{\theta} \cdot \max\left\{-e_{i,t},\ a_{i,t}\right\}, \\ \qquad\qquad\qquad\qquad \text{if } a_{i,t} < 0 \text{ (discharging)}, \\ Q^{\theta}_{i,t} + \dfrac{\overline{e}^{\theta} \cdot \min\left\{1 - e_{i,t},\ a_{i,t}\right\}}{\eta(a_{i,t})}, \\ \qquad\qquad\qquad\qquad \text{if } a_{i,t} \geq 0 \text{ (charging)}. \end{cases}$$

Prosumer's MDP Population Profile and Reward

- Population profie: ($H(t)$ maps from time period t to an hour of the day h)

$$p_t^{I^\theta}(S, A, H(t)) = \frac{1}{I^\theta} \sum_{i=1}^{I^\theta} \mathbb{I}_{\{s_{i,t} \in S\}} \times \mathbb{I}_{\{a_{i,t} \in A\}}$$

Let $p_h^{\infty,\theta}$ be the limit as $I^\theta, t \to \infty$ for all θ and $h \in \{1, \ldots, 24\}$. This limit represents a probability distribution over the joint state and action space.

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- One-stage Reward:

$$R_{i,t}^\theta(s_{i,t}, a_{i,t}, q_{i,t}^\theta | p_{H(t)}^\infty) = -\widehat{P}_t^{n(\theta)}(p_{H(t)}^\infty) \times b_{i,t}^\theta(s_{i,t}, a_{i,t}, Q_{i,t}^\theta),$$

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- Value Function and Bellman Equation:

$$V_h^{\pi^\theta}(s_h, p^\infty) = \mathbb{E}\left[\sum_{t=h}^{\infty} \beta^{t-1} R_t^\theta(s_t, a_t, Q_t^\theta | p_t^\infty) \; a_t \sim \pi_{h=H(t)}^\theta, \; s_h, \; p^\infty\right].$$

$$V_h^{\pi^{\theta^*}}(s, p^\infty) = \max_{a \in \mathcal{A}} \left[\widehat{R}^\theta(s, a | p^\infty) + \beta V_{h+1}^{\pi^{\theta^*}}(E(s, a), p^\infty)\right],$$

Single-valuedness of a Prosumer's Optimal Policy

- Optimal policy mapping:

$$\Pi_h^{\theta^*}(s, p^\infty) = \arg\max_{a \in \mathcal{A}} \left[\widehat{R}^\theta(s, a | p^\infty) + \beta V_{h+1}^{\pi^{\theta^*}} (E(s, a), p^\infty) \right].$$

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- Single-valuedness and continuity:

Lemma. Under the LICQ Assumption, for an agent of type θ, the optimal stationary policy mapping $\Pi^{\theta^*}(s,\ p^\infty)$ is single-valued and continuous with respect to $(s,\ p^\infty)$.

Part III – Mean Field Equilibrium and Algorithm

MFE Definition

- Definition: A collection of stationary strategy $\pi^* := \left[[\pi_1^{\theta^*}]_{\theta\in\Theta}, \cdots, [\pi_{24}^{\theta^*}]_{\theta\in\Theta}\right]$ and a population profile $p^\infty := \left[[p_1^{\infty,\theta}]_{\theta\in\Theta}, \cdots, [p_{24}^{\infty,\theta}]_{\theta\in\Theta}\right] \in \mathcal{P}(\mathcal{S})^{|\Theta|\times 24}$ constitute an MFE if for each $\theta \in \Theta$ and $h = 1, \ldots, 24$, the following two conditions hold:
  - Optimality: for a given state $s \in \mathcal{S}$, $\pi_h^{\theta^*} \in \Pi_h^{\theta^*}(s, p^\infty)$ as defined in the optimal policy mapping.
  - Consistency: for all $S \times A \in \mathcal{B}(\mathcal{S}) \times \mathcal{B}(\mathcal{A})$, where $\mathcal{B}(\cdot)$ is the Borel algebra of the corresponding set, and $s \in S$,

$$p_h^{\infty,\theta}(S\times A) = \int_{S\times A} \mathbb{I}_{S\times A}\left\{E\left(s, \pi_{h-1}^{\theta^*}(s, p^\infty)\right), \pi_h^{\theta^*}\left(E\left(s, \pi_{h-1}^{\theta^*}(s, p^\infty)\right), p^\infty\right)\right.$$

- MFE Existence Under the LICQ Assumption, an MFE exists.
  Proof. Use Schauder-Tychonoff's fixed point theorem.

A Heuristic Algorithm to Update Agents' Beliefs

1. Start with a random LMP belief for each hour of day $(\widehat{LMP}_1, \ldots \widehat{LMP}_{24})$ and starting value function approximation
2. Solve the Bellman equation through value iteration
3. After receiving the actual LMP for hour h on day d (denoted $LMP_{d,h}$), update the LMP belief for hour h as follows (with $\alpha \in [0.5, 1]$)

$$\widehat{LMP}_h \leftarrow \widehat{LMP}_h - \alpha \cdot (d+1)^{-0.5}(\widehat{LMP}_h - LMP_{d,h}),$$

   (If under supply or demand shocks, update the corresponding beliefs.)

4. If no regeneration, repeat Step 2 and 3; if regeneration, goto Step 1.

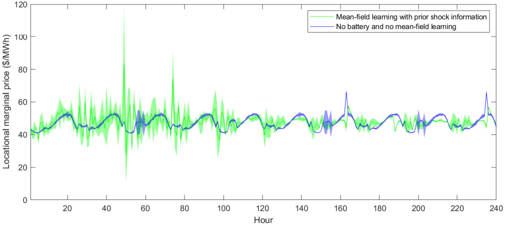
Part IV – Numerical Simulation and Results

Simulation Setup

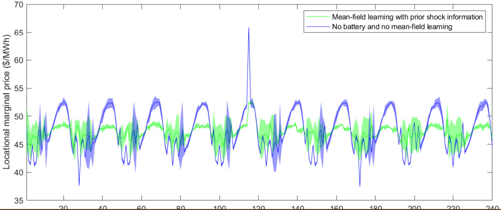
- Test case: IEEE-14 bus test case
- Agent types: prosumers and pure consumers on each bus
- Number of agents: 3,000 for each type on each bus (different buses have different types)
- Load shape: average hourly net demand (for prosumers) and gross load data (for pure consumers) from California Independent System Operator (CAISO); random noise on actual load
- Grid-level wind: fixed wind profiles at different location with random noise
- Power plants: one at each bus with a quadratic cost function from [Krishnamurthy et al., 2015]
- Simulation length: 100 days

Simulation Results LMPs

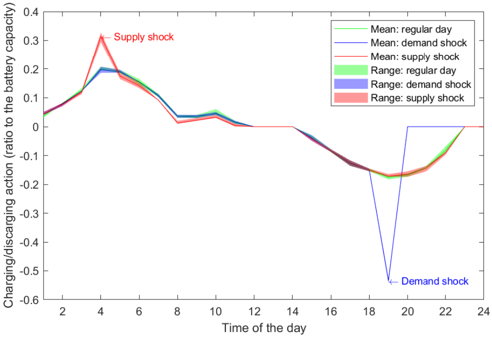
The FIRST 10 days



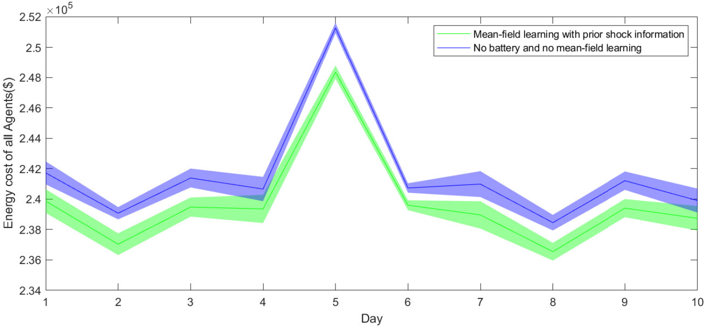
The LAST 10 days



Simulation Results   Battery Charging/Discharging



Simulation Results Total Costs



Thank you!

Acknowledgment: This research is partially supported by National Science Foundation grant ECCS-2129631 and the U.S. Department of Energy, Office of Electricity, under Award Number DE-OE0000921.