Coherence and Concentration in Tightly-Connected Networks

Model Reduction and Grid-Forming Freq. Shaping

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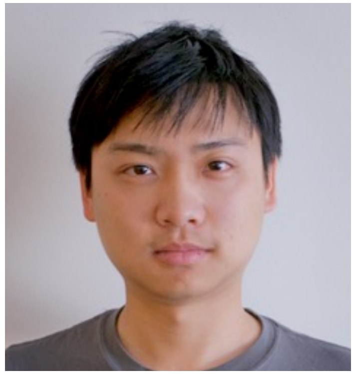
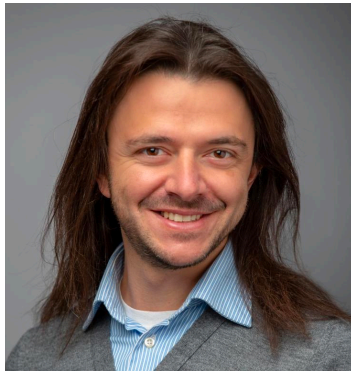
Johns Hopkins University

September 9, 2021

Acknowledgements





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NSF

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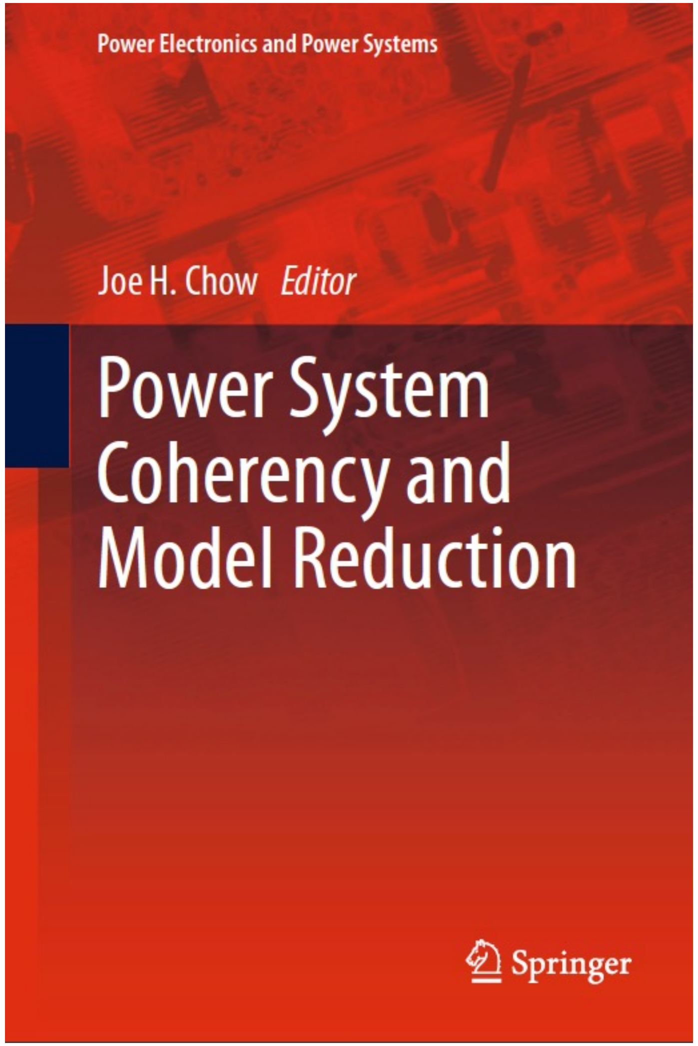
Army Research Office, Science to Shape the Future of the Army, 1951

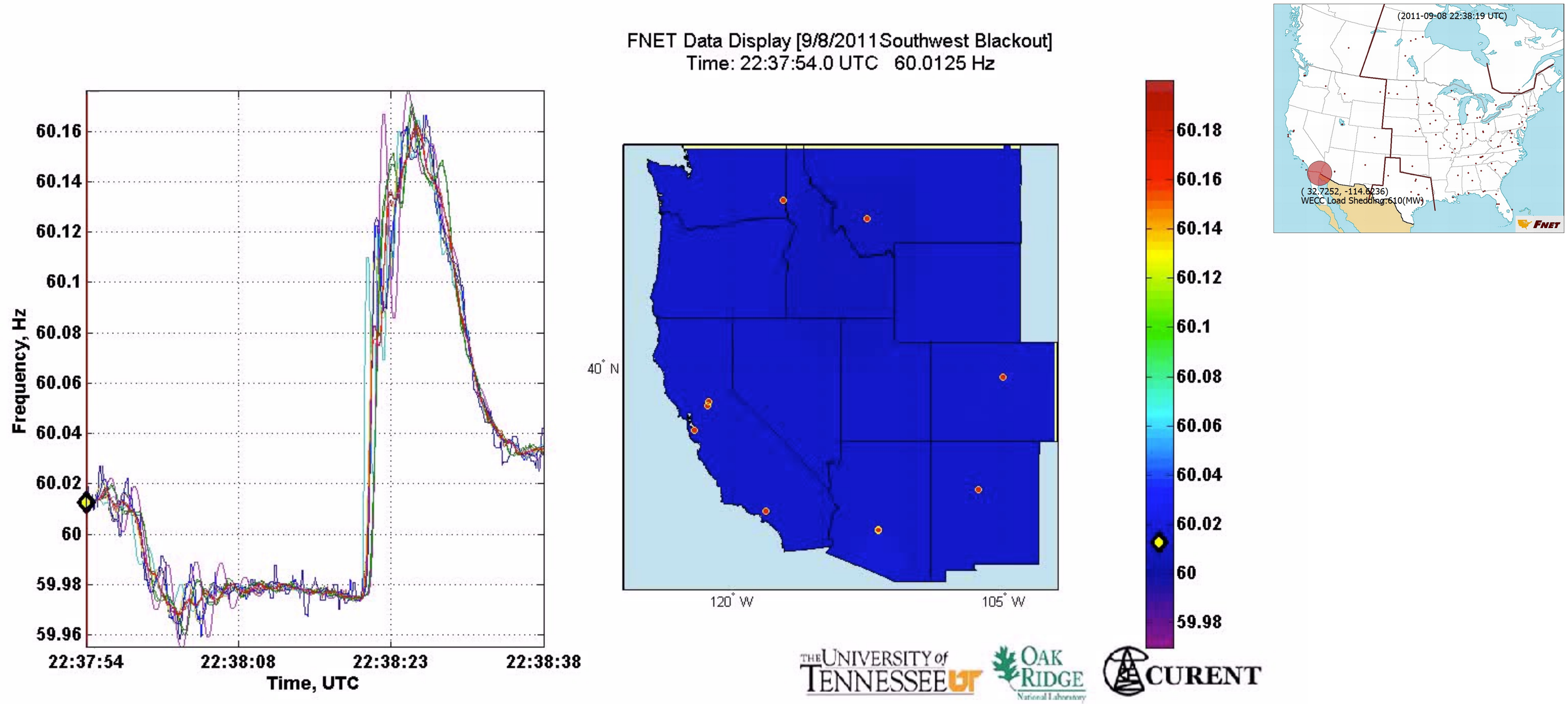
Johns Hopkins Environment, Energy, Sustainability & Health Institute

Coherence in Power Networks

- Studied since the 70s
  - Podmore, Price, Chow, Kokotovic, Verghese, Pai, Schweppe,…
- Enables aggregation/model reduction
  - Speed up transient stability analysis
- Many important questions
  - How to identify coherent modes?
  - How to accurately reduce them?
  - What is the cause?
- Many approaches
  - Timescale separations (Chow, Kokotovic,)
  - Krylov subspaces (Chaniotis, Pai ‘01)
  - Balanced truncation (Liu et al ‘09)
  - Selective Modal Analysis (Perez-Arriaga, Verghese, Schweppe ‘82)



This talk



Goal: **Characterize the coherence response from a frequency domain perspective**

Outline

- Characterization of Coherent Dynamics [Min, M ‘21]
- Reduced-Order Model of Coherent Response [Min, Paganini, M ‘21]
- Grid-forming Frequency Shaping Control [Jiang, Bernstein, Vorobev, M ‘21]

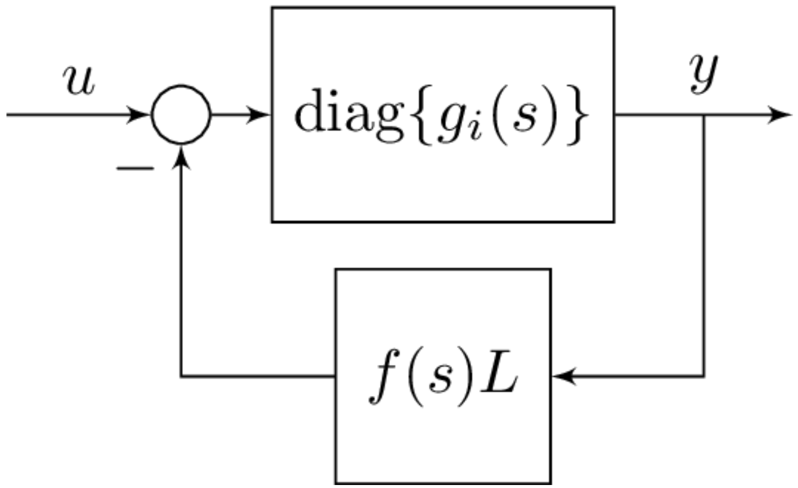
Coherence and Concentration in Tightly-Connected Networks

Hancheng Min and Enrique Mallada

ArXiv preprint: arXiv:2101.00981

Coherence in networked dynamical systems

Block Diagram:



Node dynamics: $g_i(s), i = 1, 2, \cdots, n$

Symmetric Real Network Laplacian: L

$$L = V\Lambda V^T,\ V = [\mathbb{1}/\sqrt{n}, V_\perp]$$
$$\Lambda = \mathrm{diag}\{0, \lambda_2(L), \cdots, \lambda_n(L)\}$$

Coupling dynamics: $f(s)$

Examples:

- Consensus Networks:

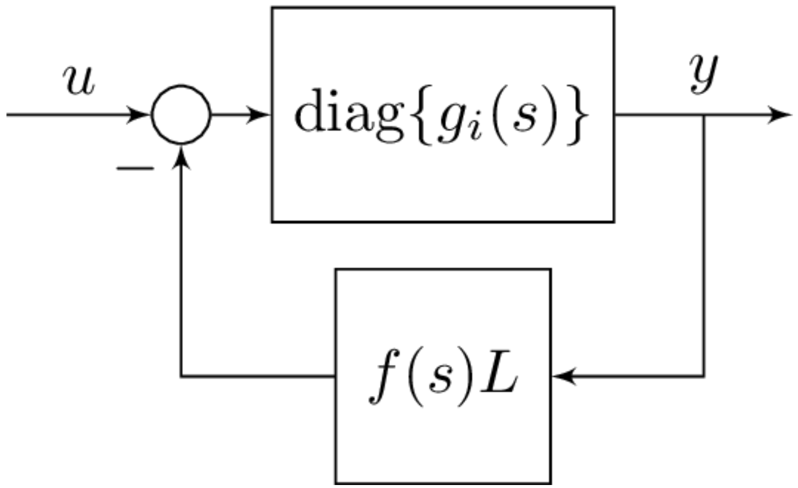
$$g_i(s) = \frac{1}{s}$$
$$f(s) = 1$$

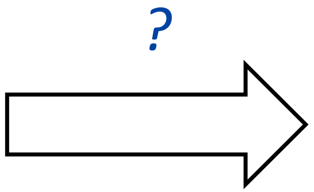
- Power Networks (2nd order generator):

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$
$$f(s) = \frac{1}{s}$$

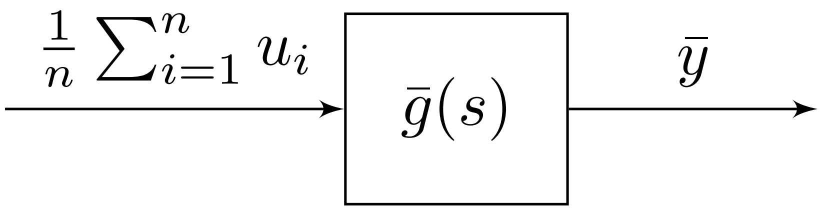
Coherence in networked dynamical systems

Block Diagram:





1.When does this network exhibit coherence?



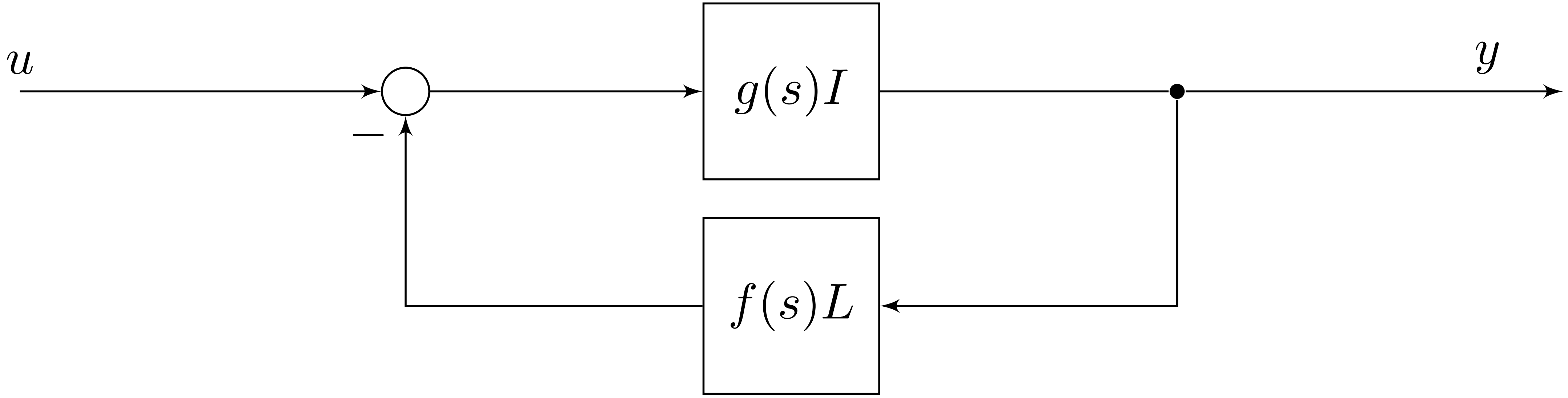
2.What is the exact ***coherent dynamics*** *of this network?*

1. Coherence can be understood as a **low rank** property the **closed-loop transfer matrix**
2. It emerges as the **effective algebraic connectivity** increases
3. The coherent dynamics is given by the **harmonic mean** of nodal dynamics

$$\bar{g}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

Network Coherence: Homogeneous Case

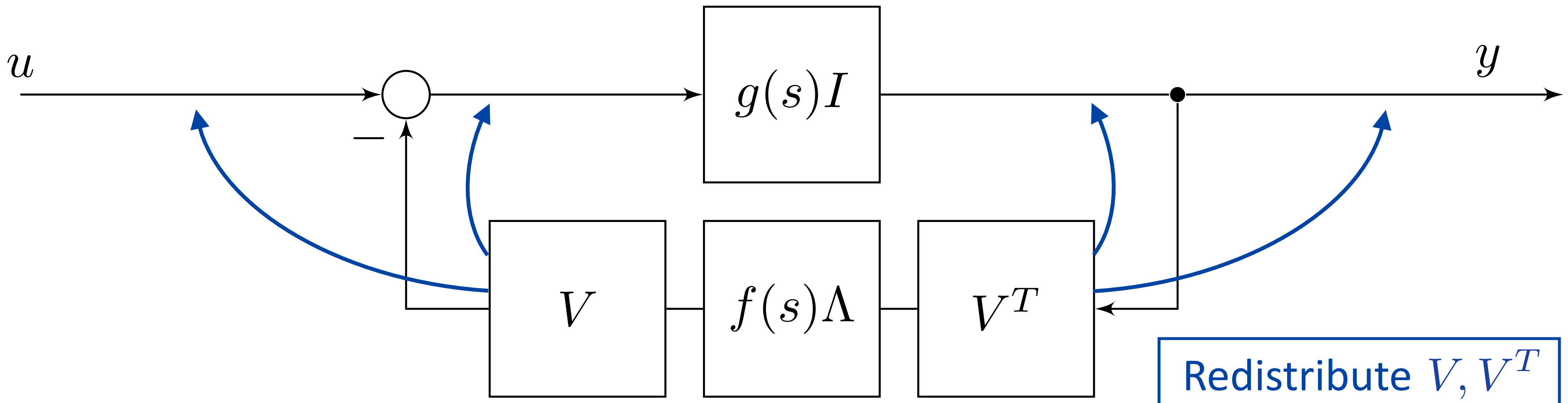
Assume homogeneity: $g_i(s) = g(s),\ i = 1, \cdots, n$



Eigendecomposition $L = V\Lambda V^T$

Network Coherence: Homogeneous Case

Assume homogeneity: $g_i(s) = g(s),\ i = 1, \cdots, n$

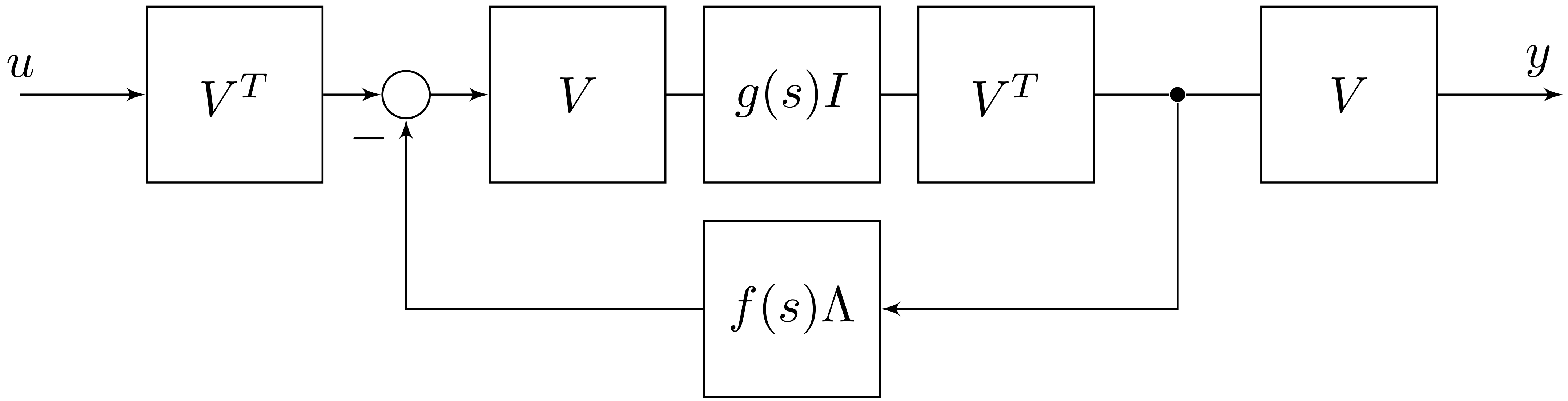


Redistribute V, V^T

Network Coherence: Homogeneous Case

Assume homogeneity: $g_i(s) = g(s),\ i = 1, \cdots, n$

Merge forward path
$V^T V = I$



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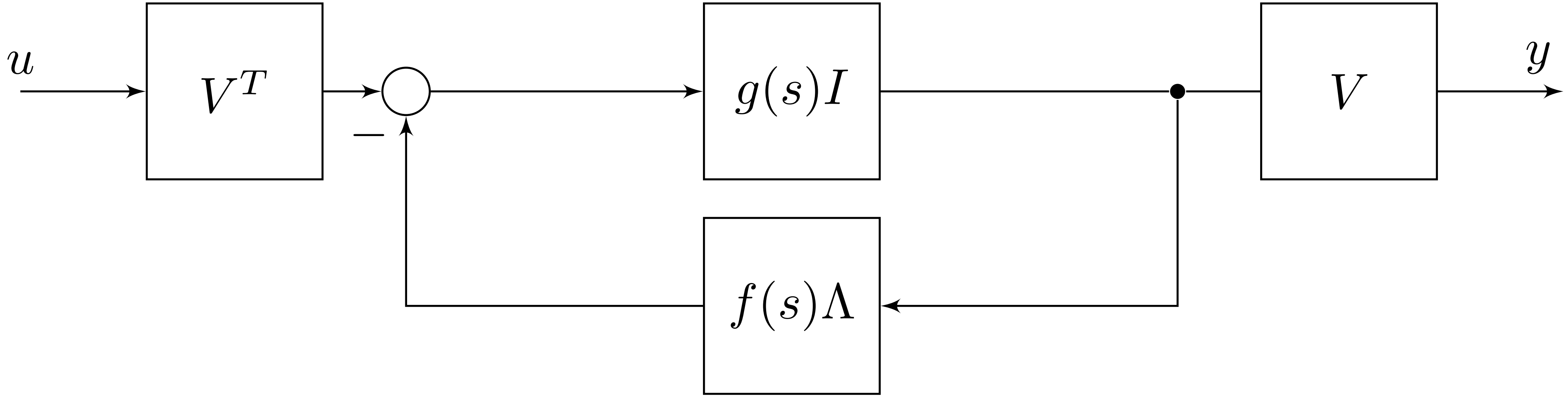

Network Coherence: Homogeneous Case

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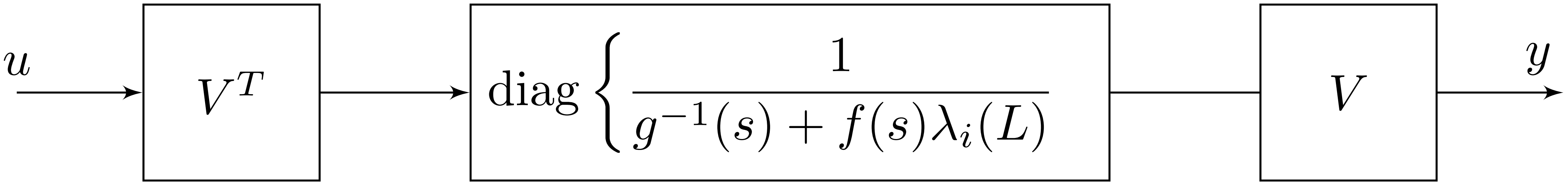


Network Coherence: Homogeneous Case

Assume homogeneity: $g_i(s) = g(s),\ i = 1, \cdots, n$

The transfer matrix from input u to output y :

$$T(s) = V \operatorname{diag}\left\{ \frac{1}{g^{-1}(s) + f(s)\lambda_i(L)} \right\}_{i=1}^{n} V^T$$

$$\boxed{V = [\mathbb{1}/\sqrt{n}, V_\perp],\ \lambda_1(L) = 0}$$

$$T(s) = \boxed{\frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T} + \boxed{V_\perp \operatorname{diag}\left\{ \frac{1}{g^{-1}(s) + f(s)\lambda_i(L)} \right\}_{i=2}^{n} V_\perp^T}$$

Coherent dynamics
independent of the network structure

Dynamics dependent of the network structure

Network Coherence: Homogeneous Case

$$T(s) = \frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T + V_\perp \mathrm{diag}\left\{ \frac{1}{g^{-1}(s) + f(s)\lambda_i(L)} \right\} V^T$$

The effect of non-coherent dynamics vanishes as:

- The **algebraic connectivity** $\lambda_2(L)$ of the network increases

  For almost any $s_0 \in \mathbb{C}$

  $$\lim_{\lambda_2(L) \to +\infty} \left\| T(s_0) - \frac{1}{n} g(s_0) \mathbb{1}\mathbb{1}^T \right\| = 0$$

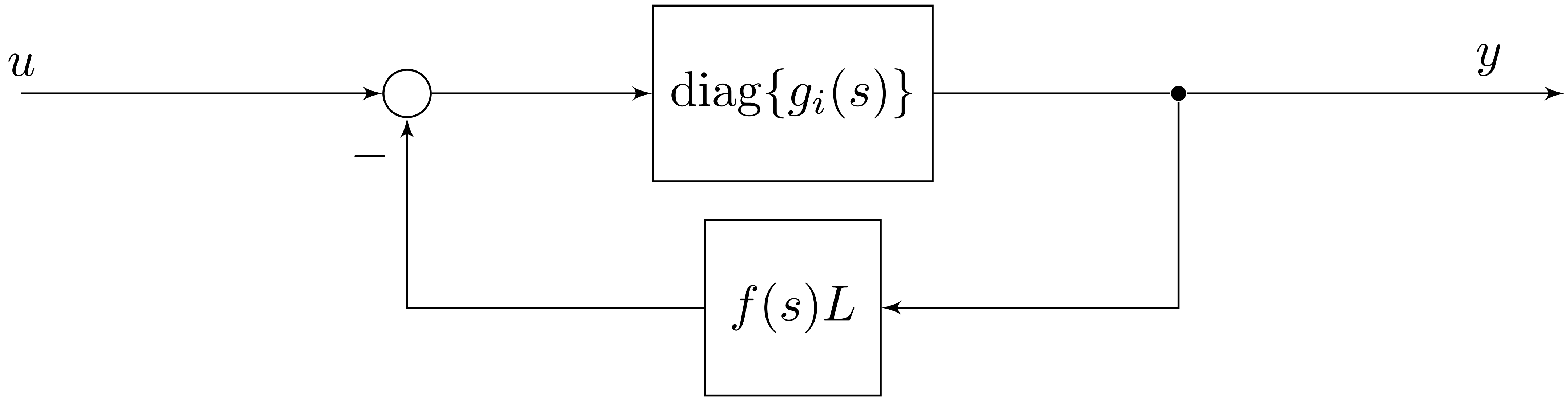
- The point of interest gets close to a **pole** of $f(s)$

  For $s_0 \in \mathbb{C}$, a pole of $f(s)$

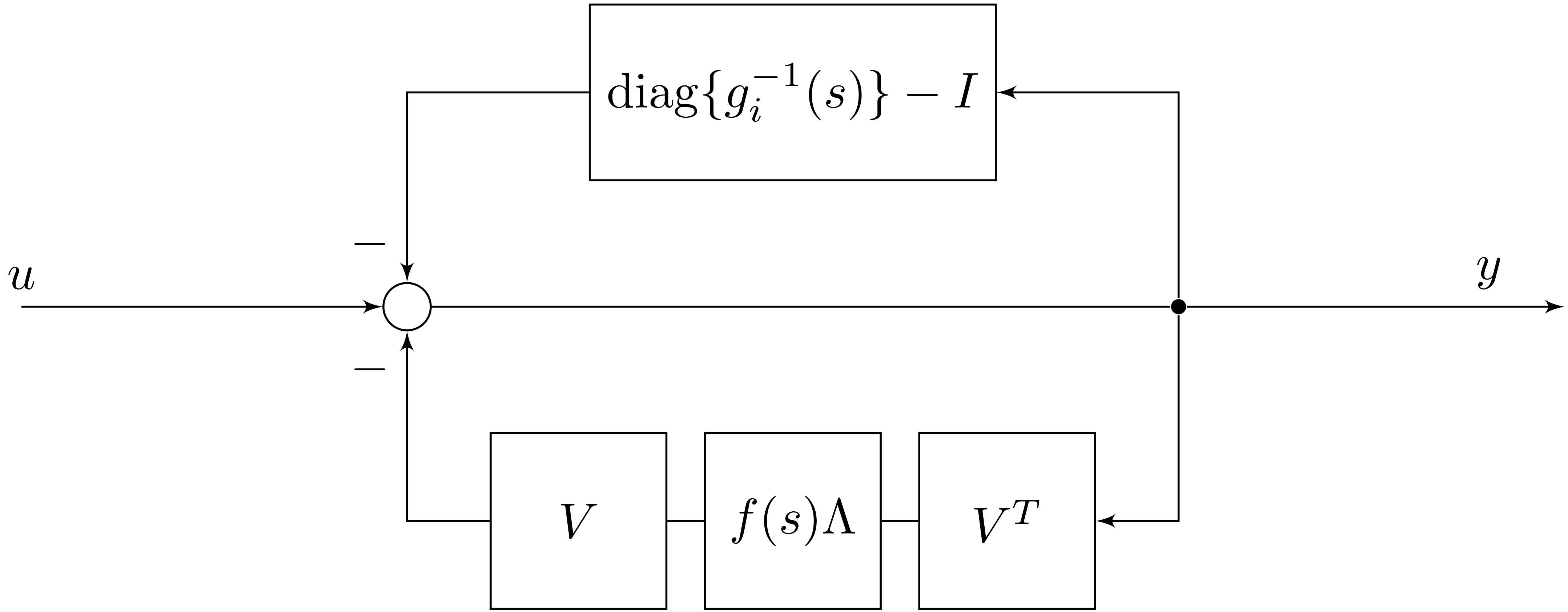
  $$\lim_{s \to s_0} \left\| T(s) - \frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T \right\| = 0$$

Our **frequency-dependent** coherence measure $\left\| T(s) - \frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T \right\|$ is controlled by the **effective algebraic connectivity** $|f(s)|\lambda_2(L)$

Network Coherence: Heterogeneous Case



Network Coherence: Heterogeneous Case



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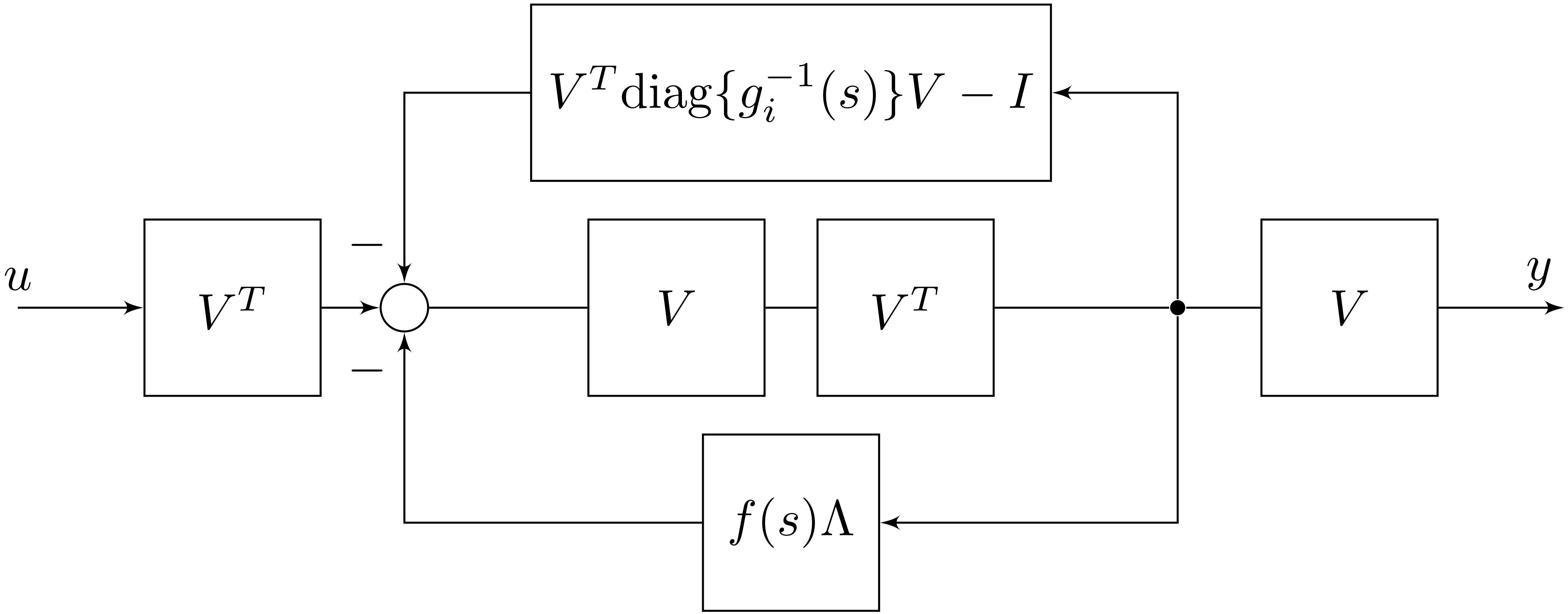
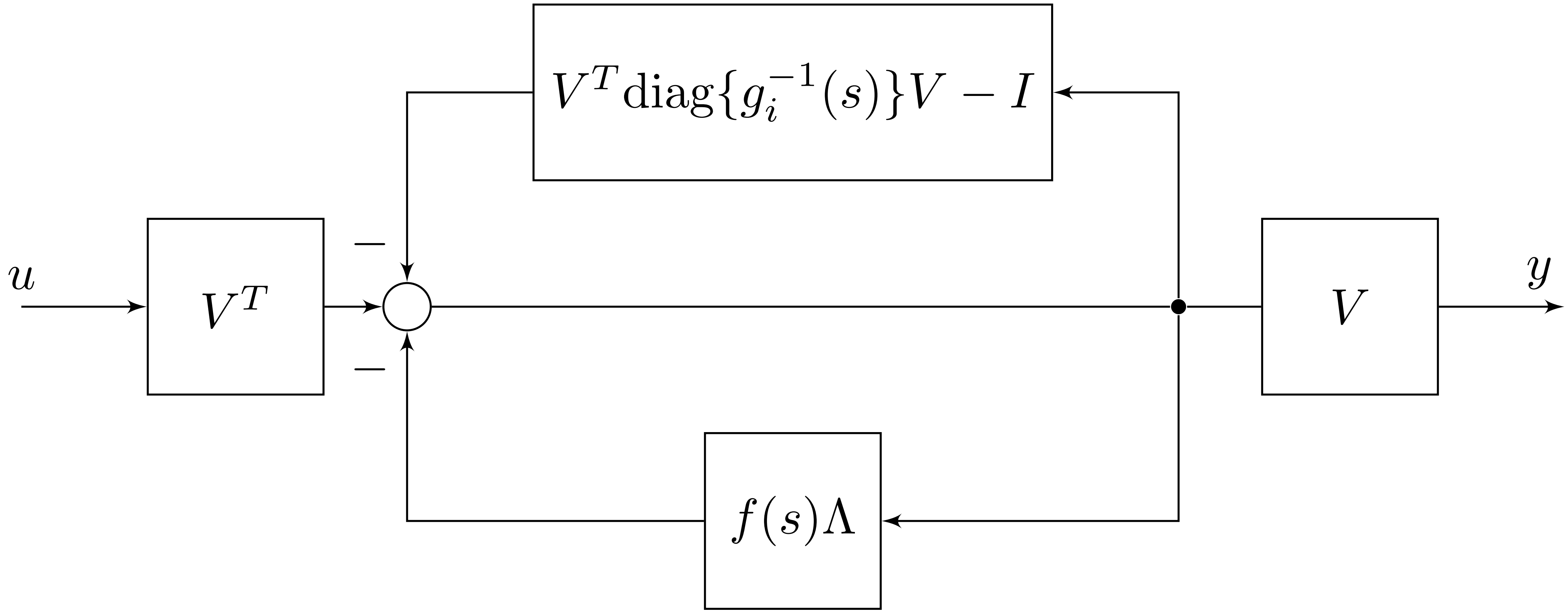
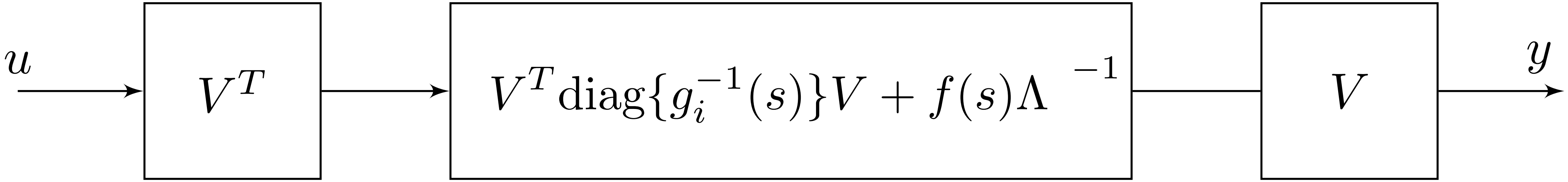


Network Coherence: Heterogeneous Case



Network Coherence: Heterogeneous Case



The transfer matrix from input u to output y :

$$T(s) = V \;\; V^T \mathrm{diag}\{g_i^{-1}(s)\} V + f(s)\Lambda \;\; ^{-1} V^T$$

Network Coherence: Heterogeneous Case

The transfer matrix from input u to output y :

$$T(s) = V \quad V^T \mathrm{diag}\{g_i \ ^1(s)\}V + f(s)\Lambda \quad ^1 V^T$$

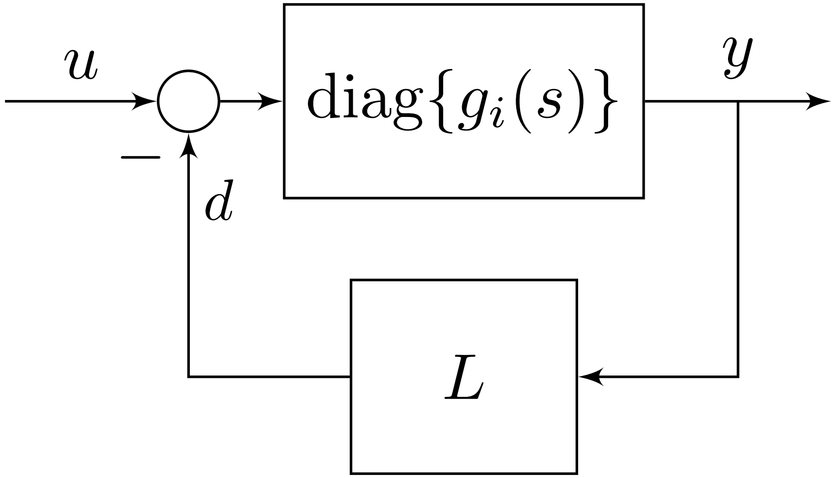
$$T(s) = \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T + N(s)$$

Coherent Dynamics?

Network dependent?

Informed guess for coherent dynamics: $\bar{g}(s)$

Block Diagram:



Dynamics for node i

$$y_i(s) = g_i(s)(u_i(s) - d_i(s)),\ i = 1, \cdots, n$$

Assume all nodes output are **identical** as the result of **coherence**

$$y_i(s) = \bar{y}(s)$$

$$g_i^{-1}(s)\bar{y}(s) = u_i(s) - d_i(s),\ i = 1, \cdots, n$$

Average equations from $i = 1$ to n:

$$\mathbb{1}^T L = \mathbb{0}$$

$$\left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)\bar{y}(s) = \frac{1}{n}\sum_{i=1}^{n} u_i(s) - \frac{1}{n}\sum_{i=1}^{n} d_i(s)$$

=0

Coherent Dynamics:

$$\bar{y}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1} \frac{1}{n}\sum_{i=1}^{n} u_i(s)$$

$$\bar{g}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

Harmonic mean of all $g_i(s)$

Network Coherence: Heterogeneous Case

$$T(s) = \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T + \boxed{T(s) - \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T} \qquad \bar{g}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

The effect of non-coherent dynamics vanishes as:

- For almost any $s_0 \in \mathbb{C}$

$$\lim_{\lambda_2(L)\to+\infty} \left\| T(s_0) - \frac{1}{n}\bar{g}(s_0)\mathbb{1}\mathbb{1}^T \right\| = 0$$

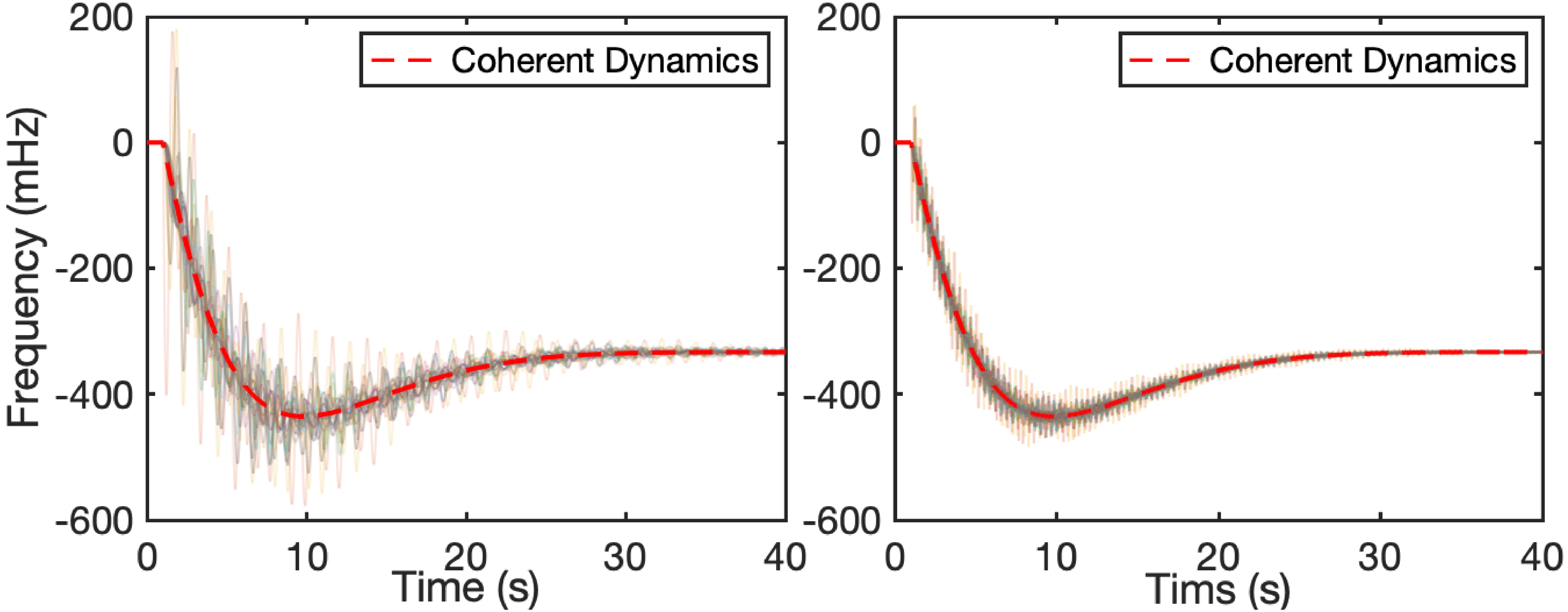
- For $s_0 \in \mathbb{C}$, a pole of $f(s)$

$$\lim_{s\to s_0} \left\| T(s) - \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T \right\| = 0$$

- Excluding zeros: the limit holds at zero, but by different convergence result
- We can further prove **uniform convergence** over a compact subset of complex plane, if it doesn't contain any zero nor pole of $\bar{g}(s)$
- Convergence of transfer matrix is **related to time-domain response** by Inverse Laplace Transform
- Extensions for random network ensembles $\bar{g}(s) = (E_w[\, g^{-1}(s,w)])^{-1}$

Effect of Network Algebraic Connectivity

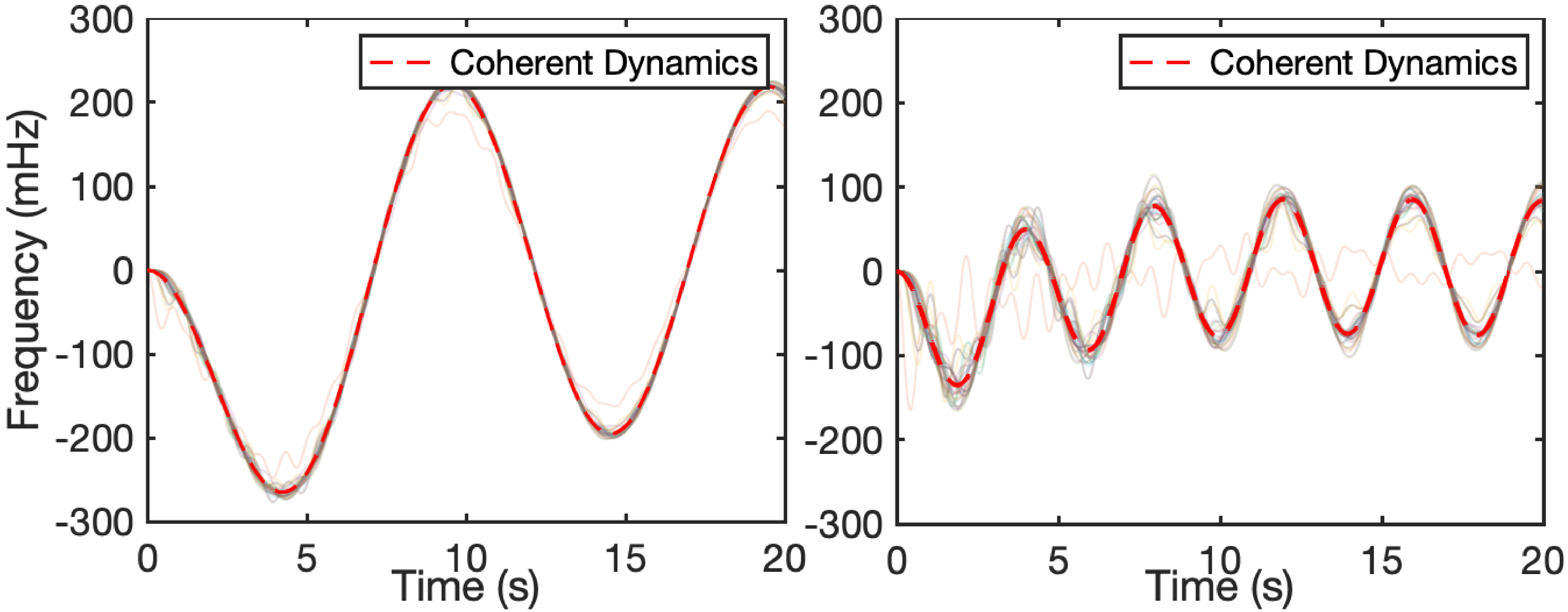
$\lambda_2(L) \uparrow$



Coherent dynamics acts as a more accurate version of the Center of Inertia (CoI)

Sinusoidal Disturbances: $\sin(\boldsymbol{\omega_d t})$

$\omega_d \uparrow$



Outline

- Characterization of Coherent Dynamics [Min, M '21]
- Reduced-Order Model of Coherent Response [Min, Paganini, M '21]
- Grid-forming Frequency Shaping Control [Jiang, Bernstein, Vorobev, M '21]

Accurate Reduced-Order Models for Heterogeneous Coherent Generators

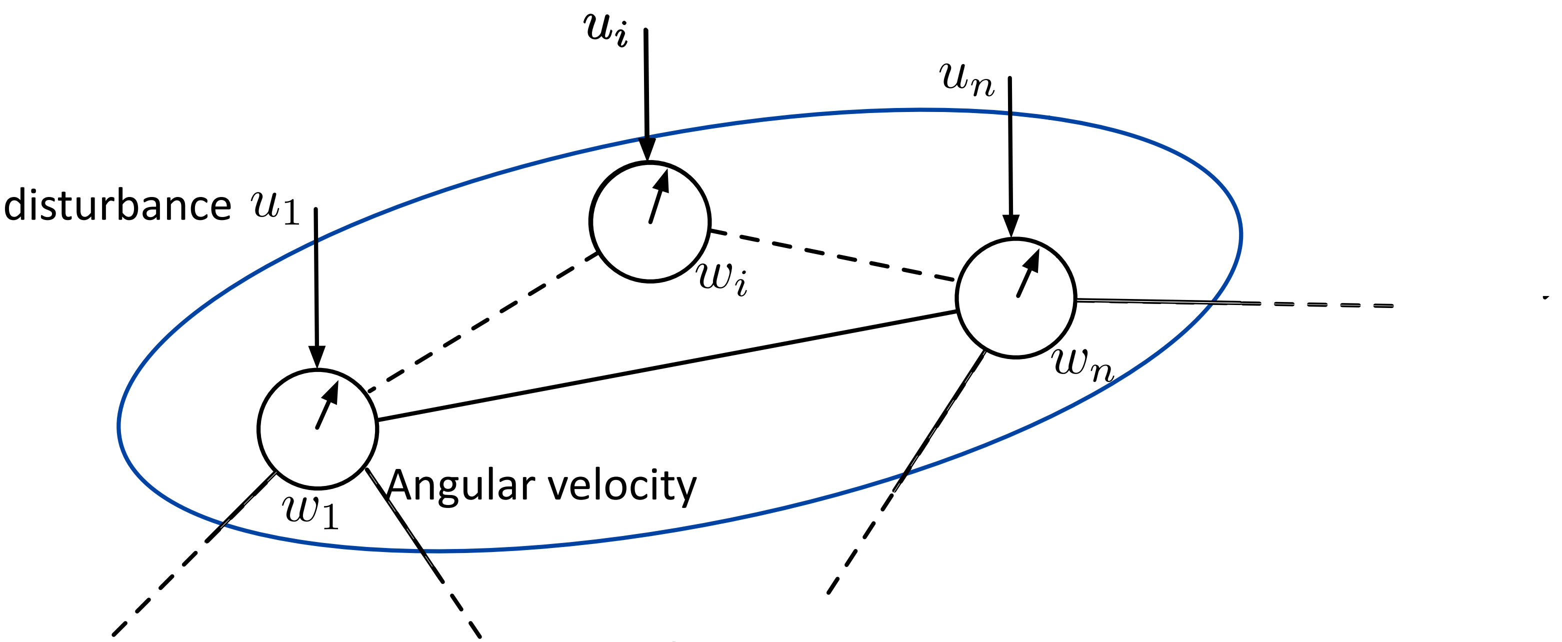
Hancheng Min, Fernando Paganini, and Enrique Mallada

IEEE Control Systems Letters, *2021*

Aggregation of Coherent Generators

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

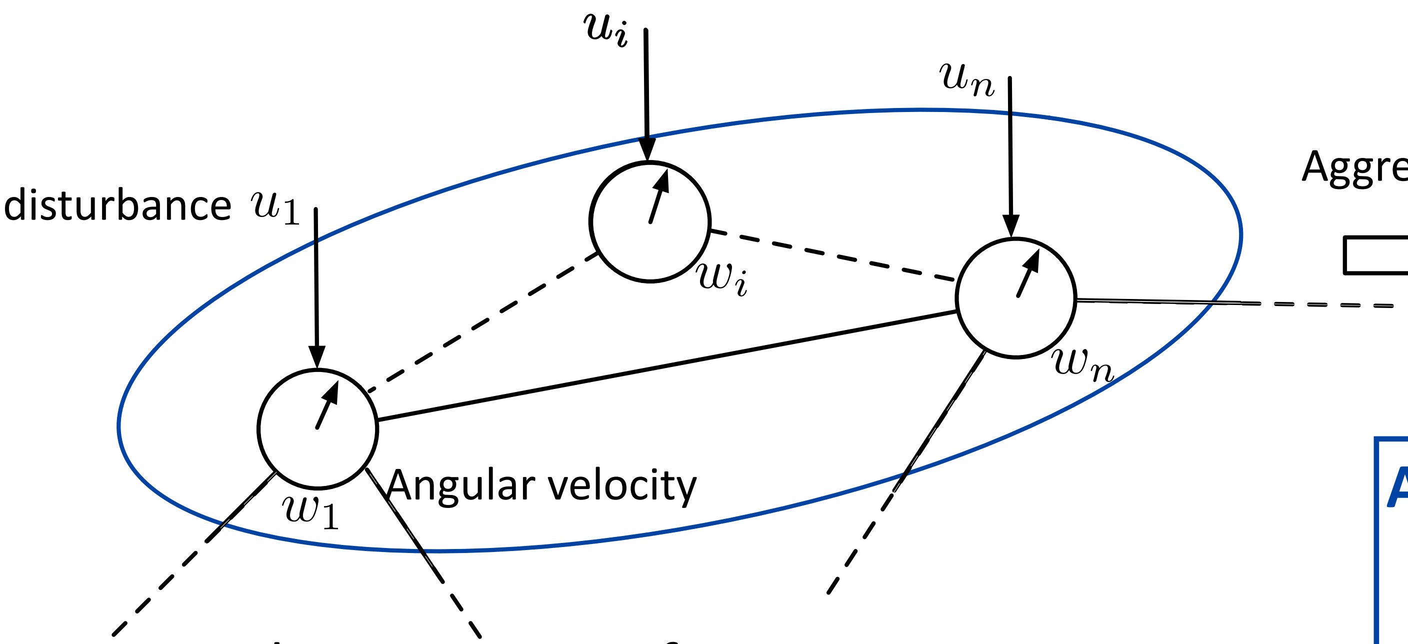
m_i: inertia
d_i: damping coefficient
r_i^{-1}: droop coefficient
τ_i: turbine time constant



coherent group of n generators

Aggregation of Coherent Generators

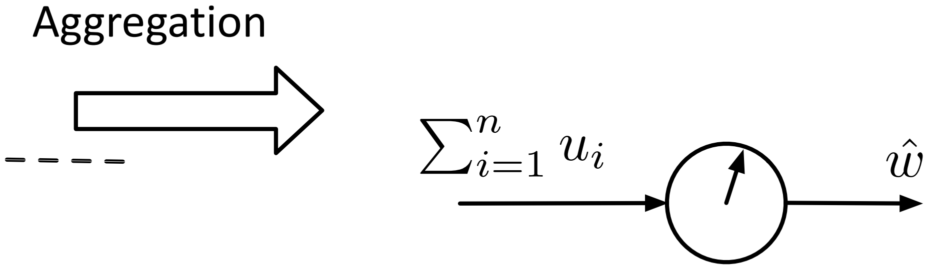
- $$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$



coherent group of n generators

Question: How to choose the different parameters of $\hat{g}(s)$?

- $$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \frac{\hat{r}^{-1}}{\hat{\tau}s+1}}$$



Answer: Use instead

$$\hat{g}(s) = \frac{1}{n}\bar{g}(s) = \left. \sum_{i=1}^{n} g_i^{-1}(s) \right)^{-1}$$

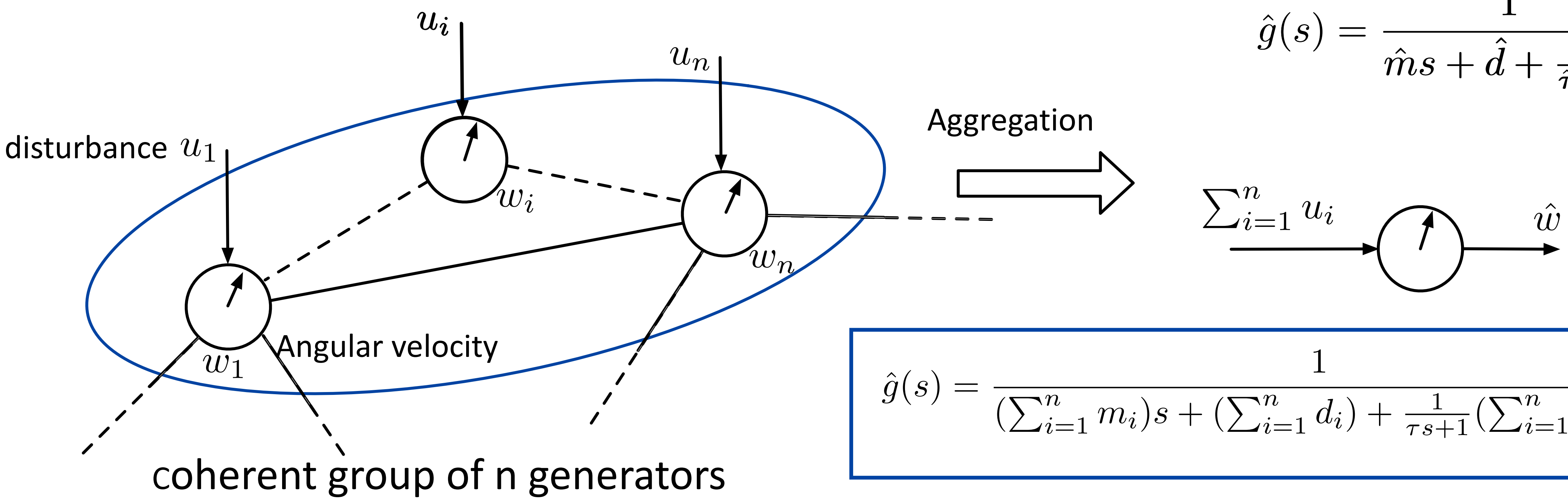
Aggregation for Homogeneous $\tau_i = \tau$

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

then $\hat{m} = \sum_{i=1}^{n} m_i, \quad \hat{d} = \sum_{i=1}^{n} d_i, \quad \hat{r}^{-1} = \sum_{i=1}^{n} r_i^{-1}$

suppose $\tau_i = \tau$

$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \frac{\hat{r}^{-1}}{\hat{\tau}s+1}}$$



$$\hat{g}(s) = \frac{1}{(\sum_{i=1}^{n} m_i)s + (\sum_{i=1}^{n} d_i) + \frac{1}{\tau s+1}(\sum_{i=1}^{n} r_i^{-1})}$$

Challenges on Aggregating Coherent Generators

For generator dynamics given by a swing model with turbine control:

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

The aggregate dynamics:

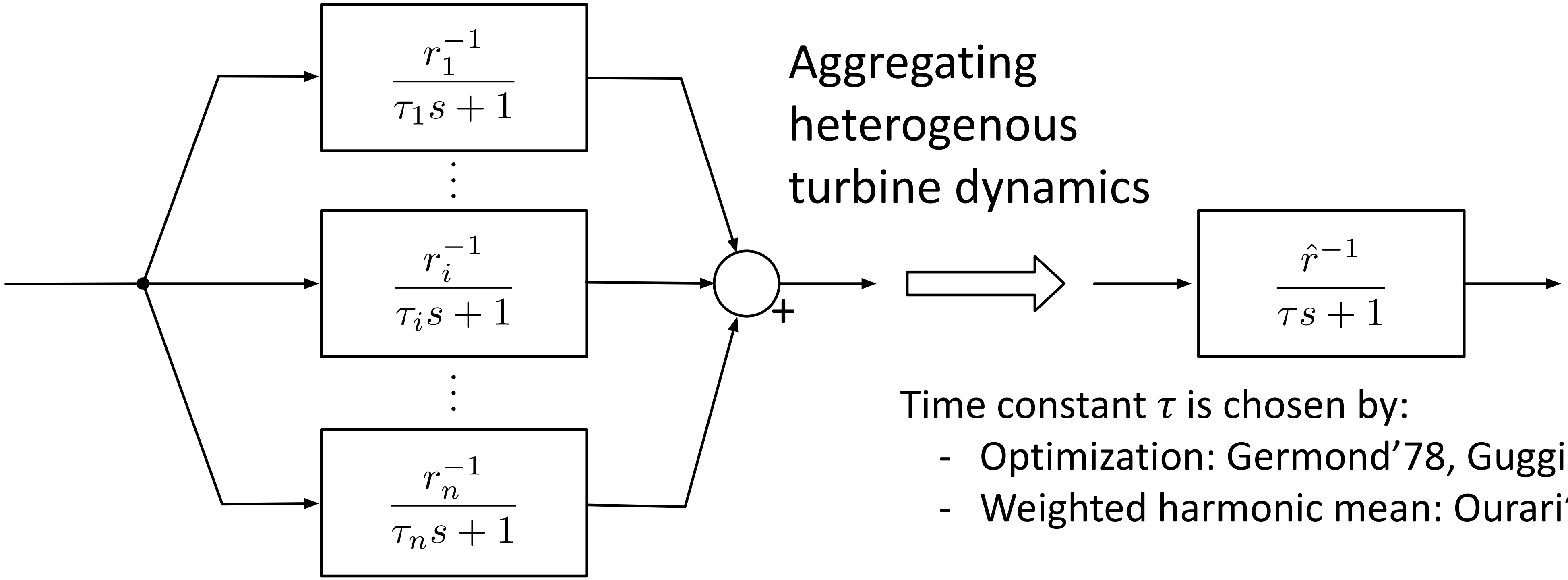
$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \boxed{\sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s + 1}}}$$

high-order if τ_i are heterogeneous

Need to find a low-order approximation of $\hat{g}(s)$

Prior Work: Aggregation for heterogeneous τ_is

When time constants are **heterogenous**:



Aggregating heterogenous turbine dynamics

Time constant τ is chosen by:

- Optimization: Germond'78, Guggilam'18
- Weighted harmonic mean: Ourari'06

Drawbacks:

- the order of overall approximation model is restricted to 2nd order
- the only "decision variable" is the time constant
- does not consider the effect of inertia or damping in the approx.

Inaccurate Approximation

Balanced Truncation

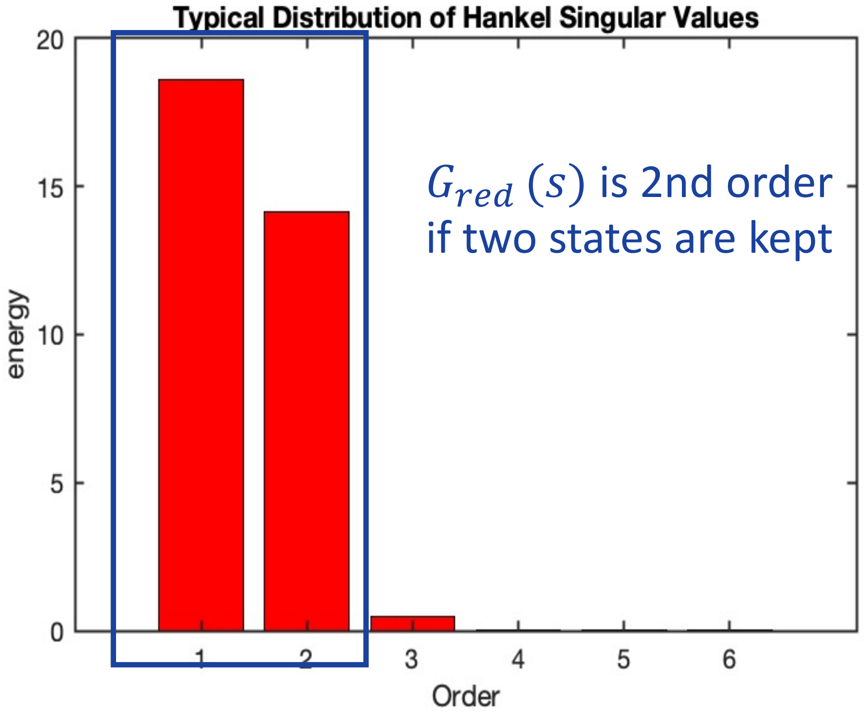
A model reduction method on stable system $G(s)$ such that:

- The reduced model $G_{red}(s)$ is stable
- The error in H_∞-norm:

$$\|G(s) - G_{red}(s)\|_{\mathcal{H}_\infty}$$

is upper bounded by a small value that depends on $G(s)$ and **the order of $G_{red}(s)$**

k-th order $G_{red}(s)$ is obtained by only keeping states of $G(s)$ associated with k largest Hankel Singular Value



There is DC gain mismatch between $G(s)$ and $G_{red}(s)$!!

Frequency Weighted Balanced Truncation

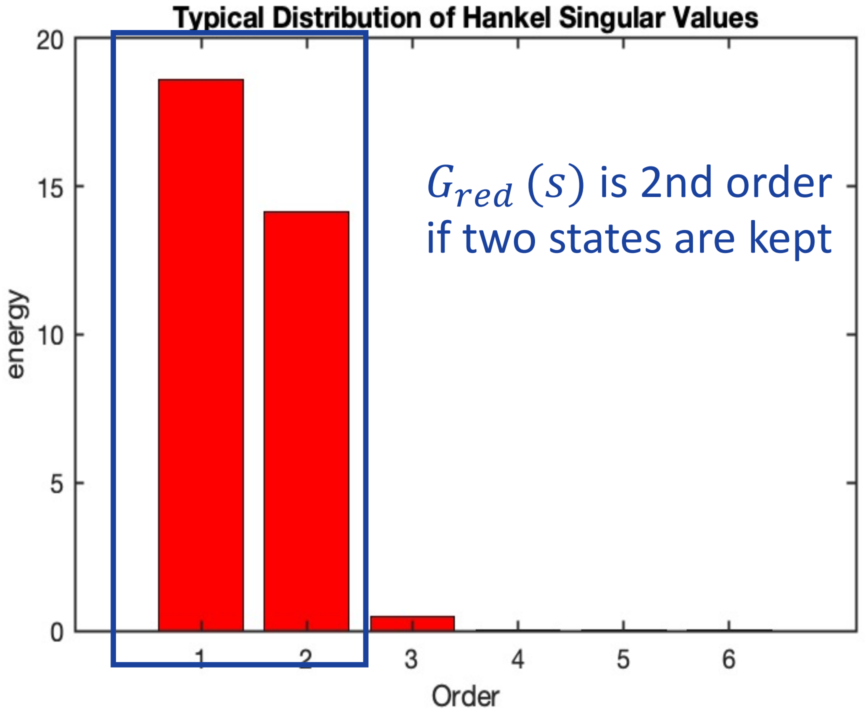
A frequency weighted model reduction method on stable system $G(s)$ such that:

- The reduced model $G_{red}(s)$ is stable
- The frequency weighted error in H_∞-norm:

$$||W(s)(G(s) - G_{red}(s)||_{\mathcal{H}_\infty}$$

is upper bounded by a small value that depends on $G(s)$ and **the order of** $G_{red}(s)$) and $W(s)$

k-th order $G_{red}(s)$ is obtained by only keeping states of $G(s)$ associated with k largest frequency weighted Hankel Singular Value



The DC gain mismatch between $G(s)$ and $G_{red}(s)$ can be made arbitrarily small weighting higher low freqs.

Aggregation Model by Frequency Weighted Balanced Truncation

Two approaches to get a k-th order reduction model of aggregate dynamics $\hat{g}(s)$:

- (k-1)-th order balanced truncation on high-order turbine dynamics

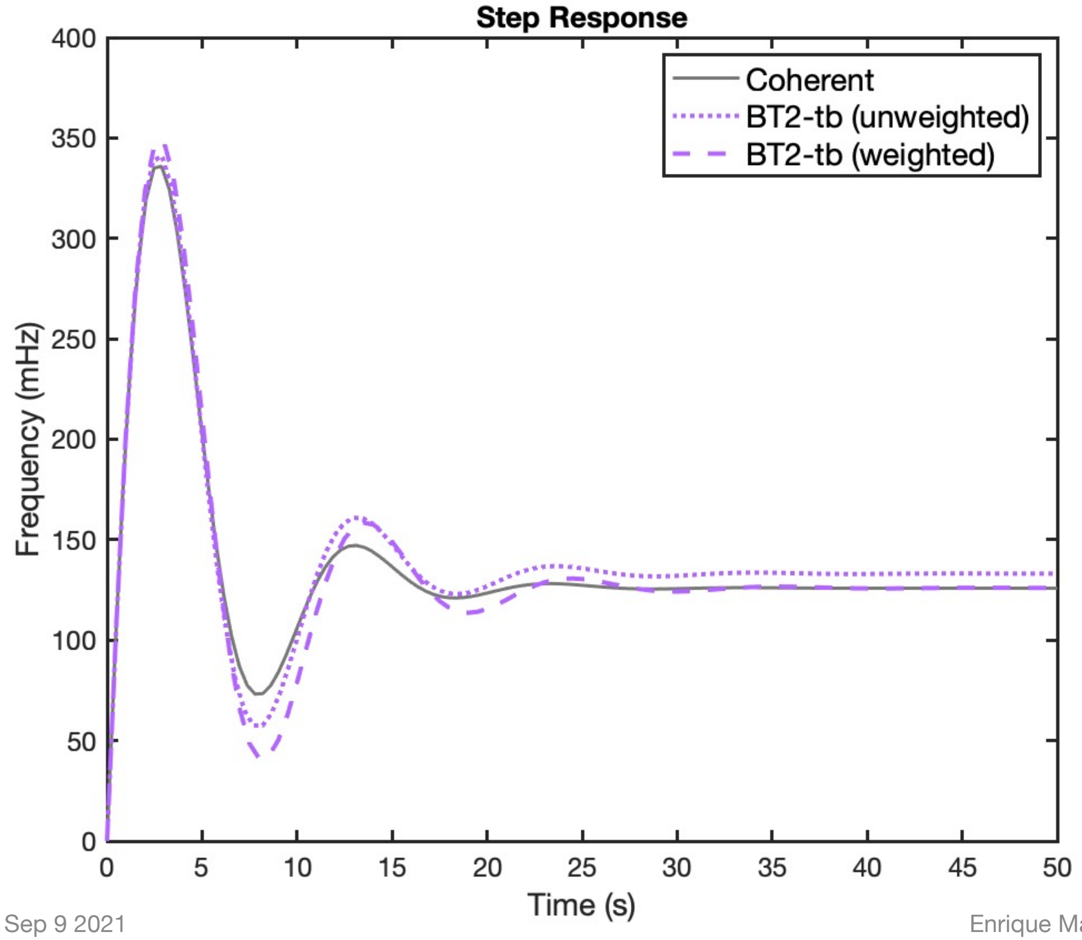
$$\tilde{g}_k^{tb}(s) = \frac{1}{\hat{m}s + \hat{d} + \boxed{\tilde{g}_{t,k-1}(s)}}$$

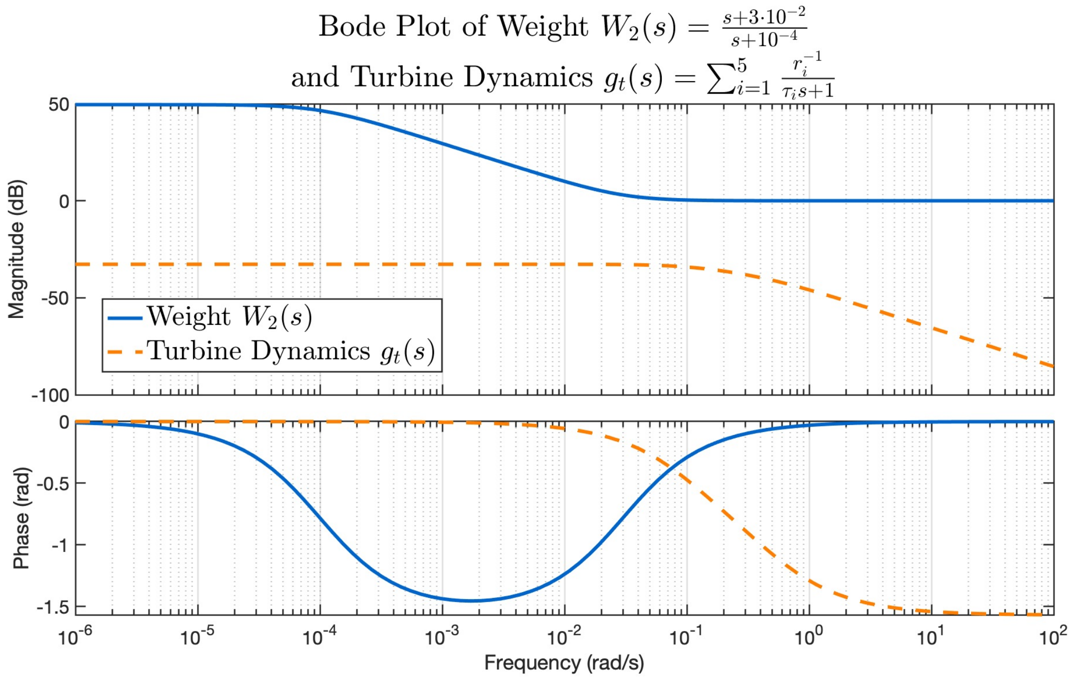
(k-1)-th reduction model on $\sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s + 1}$

- k-th order balanced truncation on closed-loop dynamics $\hat{g}(s)$

Numerical Simulation—Matching DC Gain in Balanced Truncation

Compare 2nd order model by balanced truncation on turbine dynamics with different weights: $W_1(s) = 1$ (unweighted) $W_2(s) = \frac{s + 3 \cdot 10^{-2}}{s + 10^{-4}}$ (weighted)





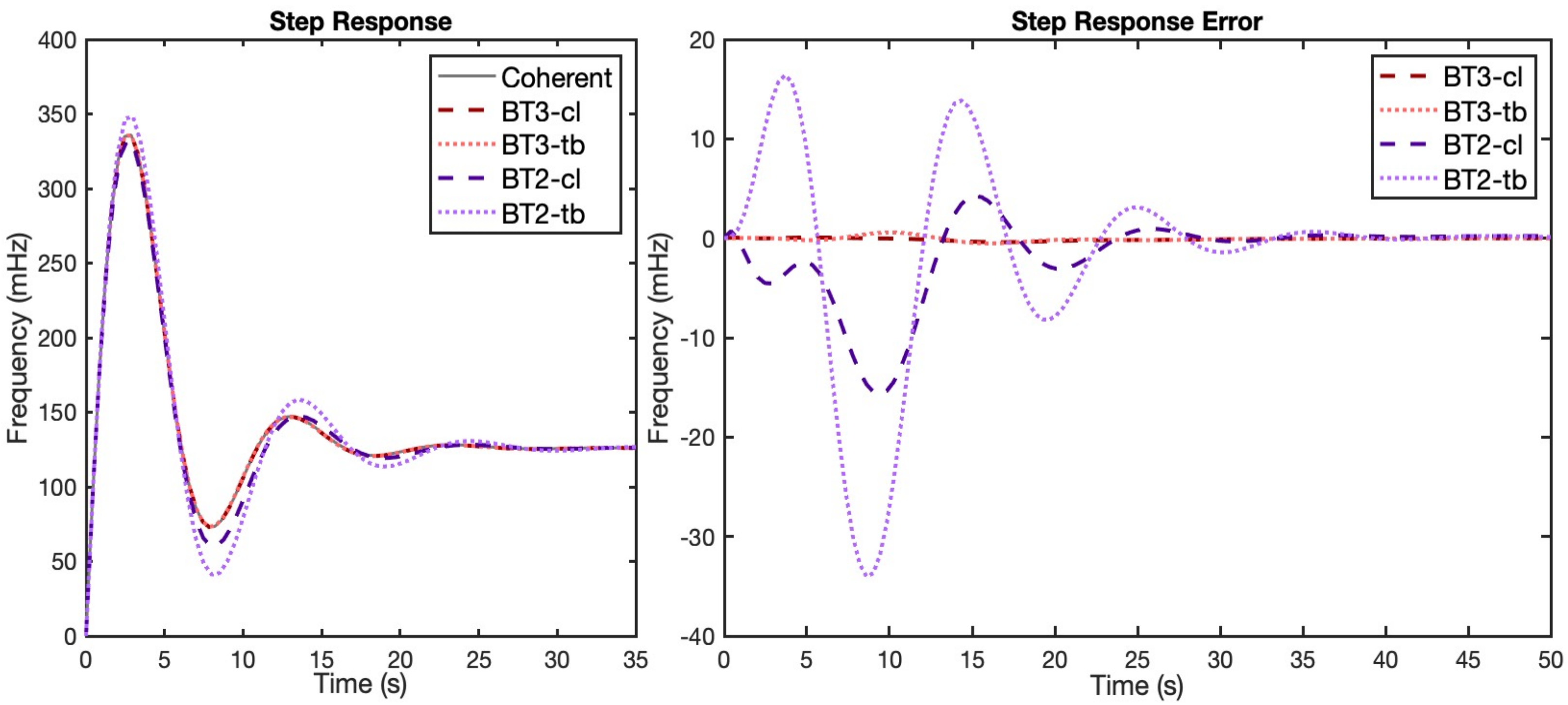
DC gain is matched by putting more weights on low frequency range

Numerical Simulation—Compare Models by Balanced Truncation

We compare the following 4 reduced order models:

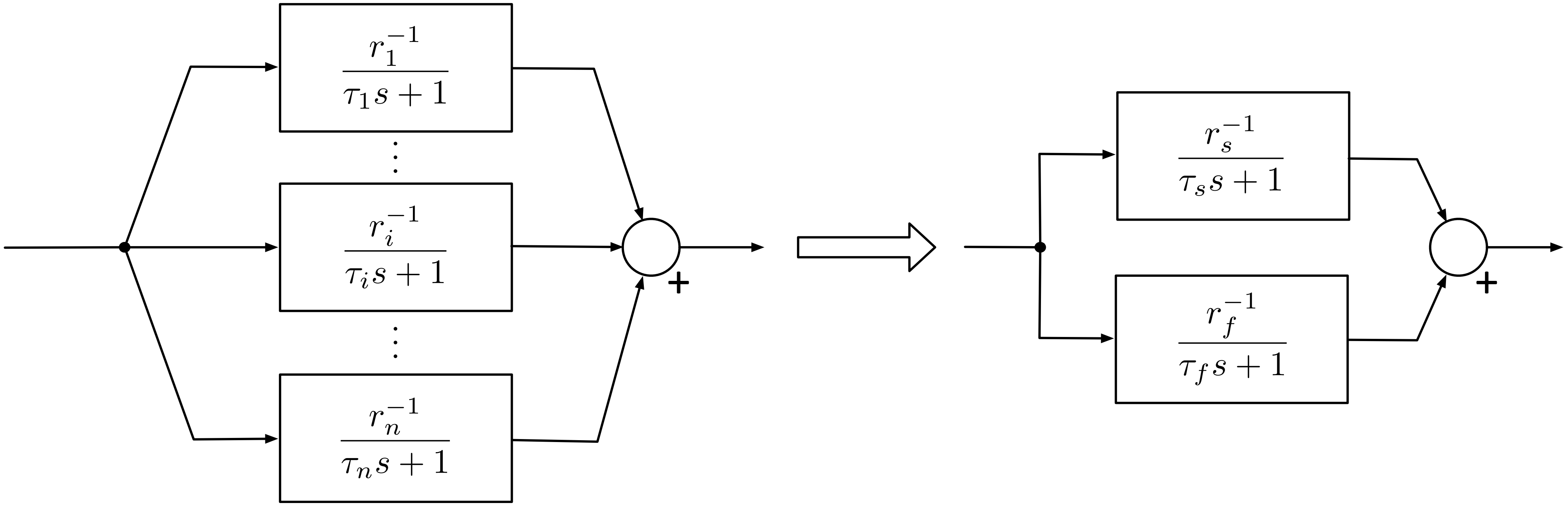
- Balanced truncation on **turbine** dynamics with weight $W_{tb}(s) = \frac{s+3\cdot 10^{-2}}{s+10^{-4}}$
  - 2nd order (BT2-tb)
  - 3rd order (BT3-tb)

- Balanced truncation on **closed-loop** dynamics with weight $W_{cl}(s) = \frac{s+8\cdot 10^{-2}}{s+10^{-4}}$
  - 2nd order (BT2-cl)
  - 3rd order (BT3-cl)



- 3rd order models are almost accurate
- balanced truncation on closed-loop is better than on turbine dynamics, given the same order

Interpretation of 3rd Order Reduced Model



- The high-order turbine dynamics can be **almost accurately** recovered by **two turbines** in parallel
- Such approximation works for aggregating even more turbines than in the test case

Outline

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- Grid-forming Frequency Shaping Control [Jiang, Bernstein, Vorobev, M '21]

Storage-Based Frequency Shaping Control

Yan Jiang, Eliza Cohn, Petr Vorobev, *Member, IEEE*, and Enrique Mallada, *Senior Member, IEEE*

[TPS 21] ***IEEE Transactions on Power Systems**, 2021*

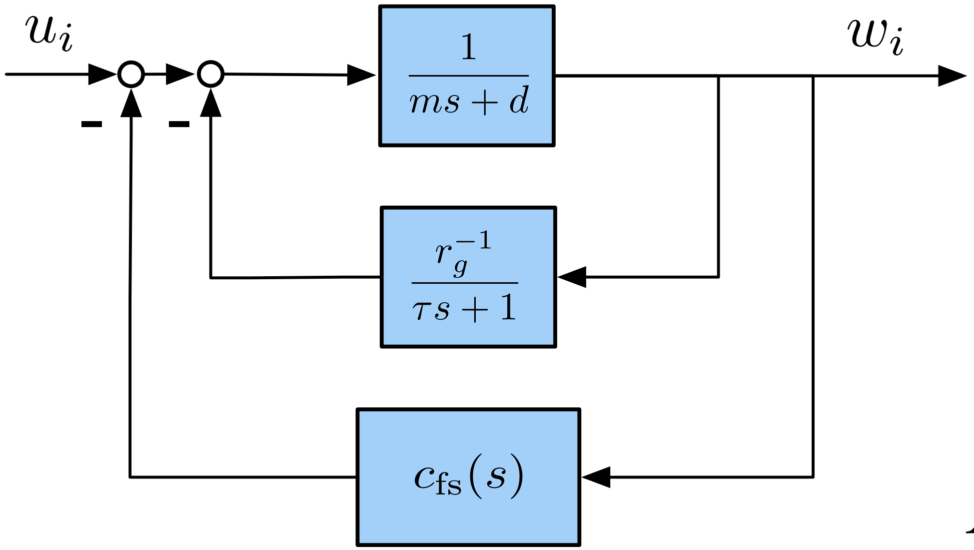
Grid-forming frequency shaping control

Yan Jiang[1], Andrey Bernstein[2], Petr Vorobev[3], and Enrique Mallada[1]

[L-CSS 21] ***IEEE Control Systems Letters,** 2021*

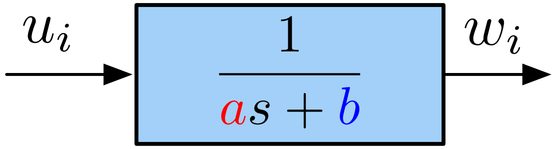
Grid-following Frequency Shaping Control

Key idea: use model matching control (at each bus)



$$c_{\text{fs}}(s) := \frac{A_1 s^2 + A_2 s + A_3}{\tau s + 1}$$

$$A_1 = \tau\,(a - m)$$
$$A_2 = b\tau + a - m$$
$$A_3 = b - r_{\text{g}} - d$$



Leads to CoI Frequency $\overline{w}$ with:

RoCoF:

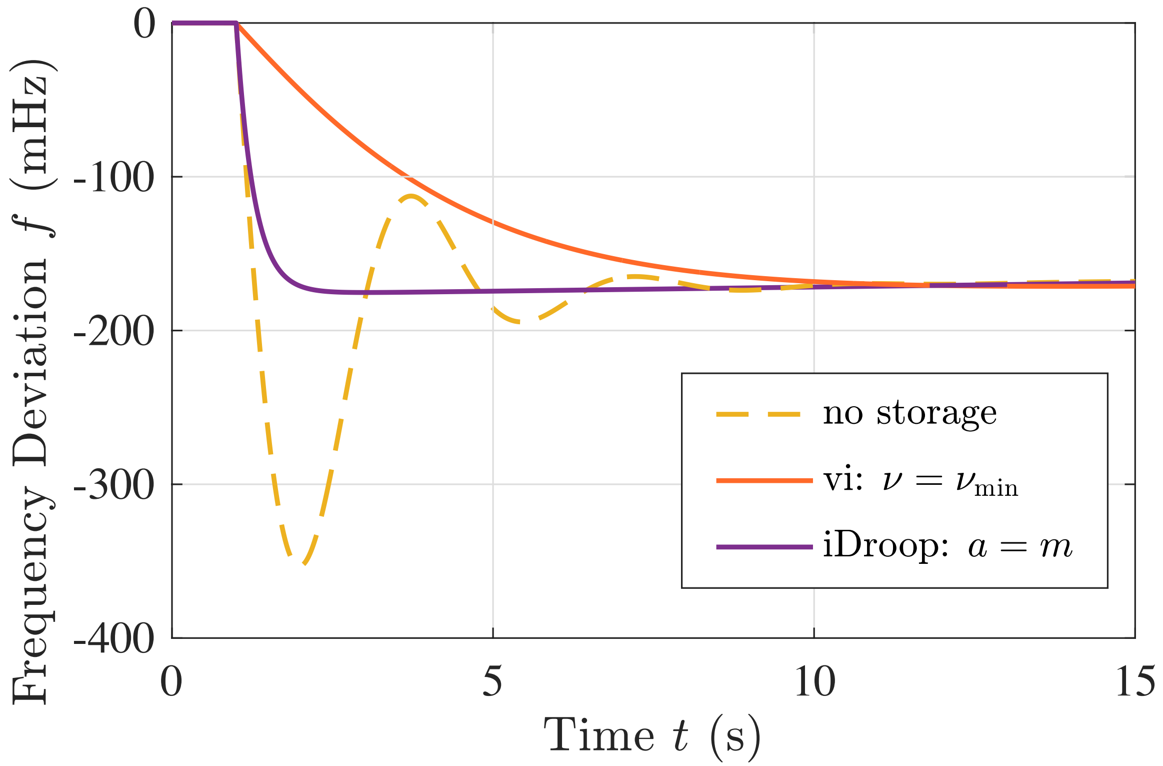
$$\|\dot{\bar{w}}\|_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i}\frac{1}{a}$$

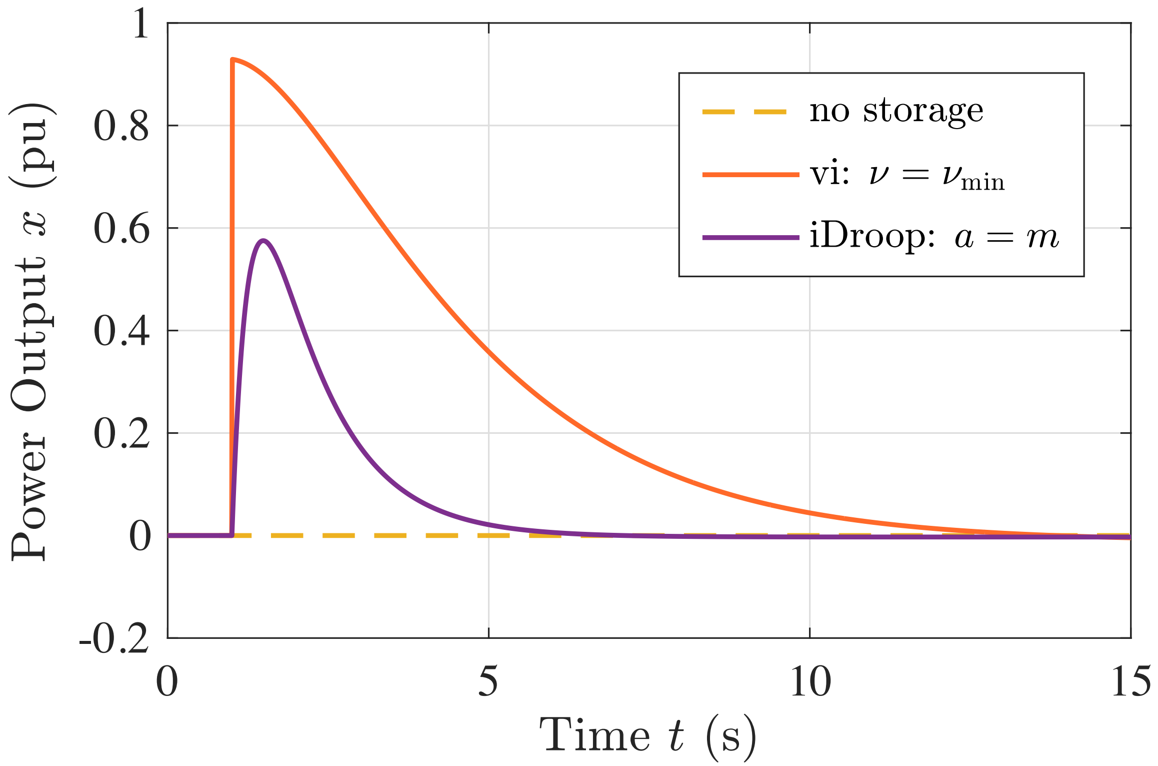
Steady-state:

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{\sum_i f_i}\frac{1}{b}$$

[TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, **IEEE Transactions on Power Systems**, under review

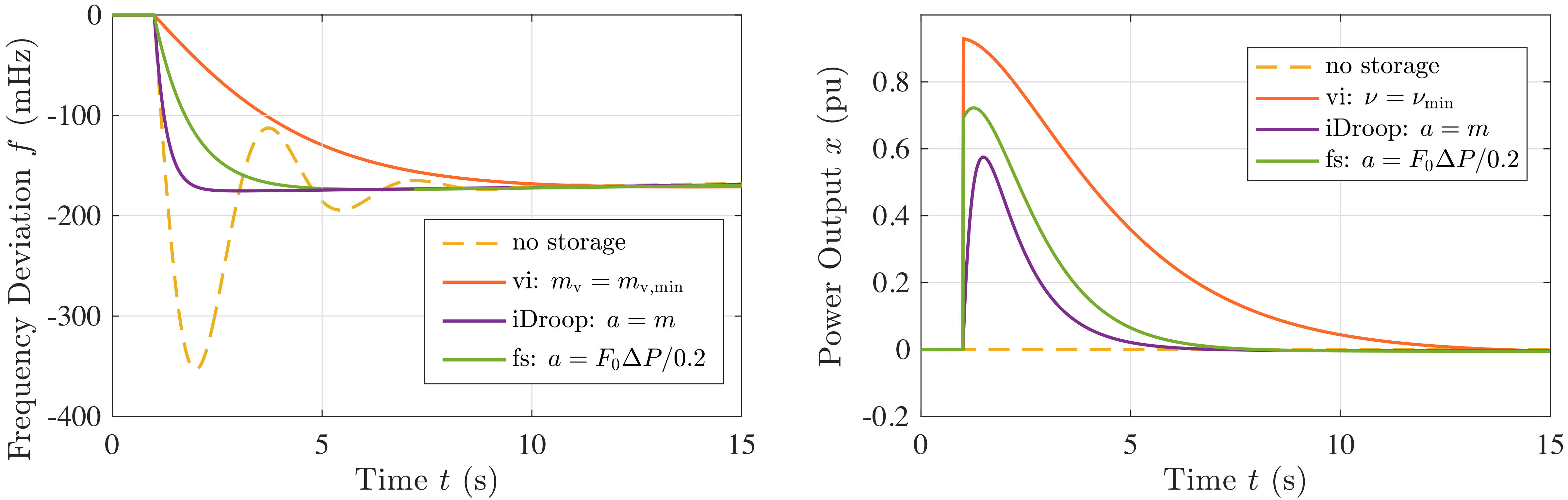
Trading off Control Effort and RoCoF





[TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, **IEEE Transactions on Power Systems**, accepted

Trading off Control Effort and RoCoF

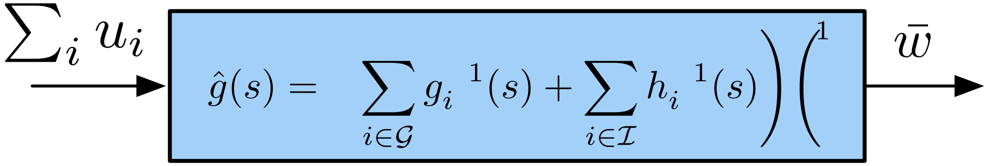


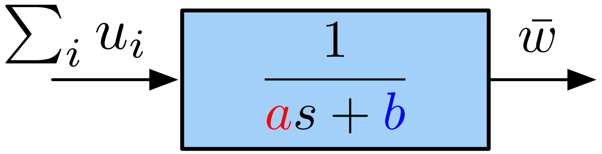
Challenge: Solution Limited to Grid-following Inverters

[TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, **IEEE Transactions on Power Systems**, accepted

Grid-forming Frequency Shaping Control

Key idea: use model matching control on coherent dynamics





Generation:

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s+1}}, \quad i \in \mathcal{G}$$

Inverters:

$$h_i(s) = \frac{1}{m_i s + d_i + c_i(s)}, \quad i \in \mathcal{I}$$

$$a := \sum_{i\in\mathcal{G}} m_i + \sum_{i\in\mathcal{I}} m_i$$

$$b := \sum_{i\in\mathcal{G}} (d_i + r_i^{-1}) + \sum_{i\in\mathcal{I}} d_i$$

$$\sum_{i\in\mathcal{I}} c_i(s) = \sum_{i\in\mathcal{G}} \frac{r_i^{-1}\tau_i s}{\tau_i s + 1}$$

RoCoF:

$$||\dot{\bar{w}}||_\infty = \frac{|\sum_i u_{0i}|}{a}$$

Steady-state:

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{b}$$

[L-CSS 21] Jiang, Bernstein, Vorobev, M, *Grid-forming frequency shaping control*, **IEEE Control Systems Letters**, 2021

Summary

- Frequency domain characterization of **coherent dynamics**, as a low rank property of the transfer function.

- **Coherence is** a **frequency dependent** property:
    - Effective algebraic connectivity $f(s)\lambda_2(L)$
    - Disturbance frequency spectrum

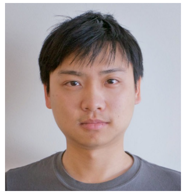
- We use frequency **weighted balanced truncation** to suggest possible improvements to obtain accurate reduced order model of aggregated dynamics of coherent generators:
    - increase model complexity (3^{rd} order/two turbines)
    - model reduction on closed-loop dynamics

- Grid-forming Frequency Shaping Control

Thanks!

Related Publications:

- Min, M, “Coherence and Concentration in Tightly Connected Networks,” **submitted**
- Min, Paganini, M, “Accurate Reduced Order Models for Coherent Synchronous Generators,” **L-CSS 2021**
- Jiang, Bernstein, Vorobev, M, “Grid-forming Frequency Shaping Control,” **L-CSS 2021**



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