

Non-cooperative Games to Control Learned Inverter Dynamics of Distributed Energy Resources

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Sept 4, 2024

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Introduction

Microgrids and DERs' role in future energy systems



- Resiliency to extreme weather events¹
 - Single microgrid: provide access to power
 - Potential value of networked microgrids
- Support the transmission network
 - Wholesale electricity market and FERC 2222
 - Ancillary services (freq. reg., spinning reserves, capacity, etc.)

¹Image source: <http://www.snopes.com/>

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Coordination of DERs in a microgrid

- DERs can work as a *Virtual Power Plant (VPP)* to provide services to support the upper-level grid.

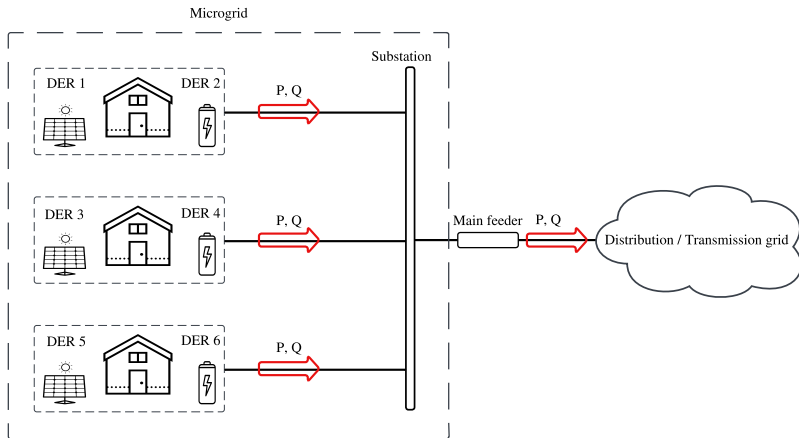


Figure: A grid-connected microgrid with DERs

Work using **optimization** techniques:

- Dall'Anese et al.² propose an online algorithm for a distribution grid to solve its ACOPF while satisfying an output power reference
- Behi et al.³ develop a bidding strategy for a VPP to maximize profits from selling load-following ancillary services, subject to customer preferences and hourly operational constraints

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Previous work: DER control for regulation services

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Benefits of optimization-based methods to control DERs:

- ✓ online implementation
- ✓ fast computation

However, they may disregard:

- × Selfish DERs, i.e., they seek to optimize their individual economic interests

Work using **non-cooperative game theory**:

- Mylvaganam et al.⁴ propose a control scheme to steer the state of a microgrid to nominal operating conditions by controlling the input impedance of storage units
- Zhang et al.⁵ develop a control scheme to coordinate DERs in an islanded MG to bring frequency deviations back to zero

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Research gap

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Past work does not:

- consider nonlinear **dynamics of inverters**,
- use **learned** state-space models that represent the dynamics of inverters, and
- implement the resulting controllers in **high-fidelity** models of inverters.

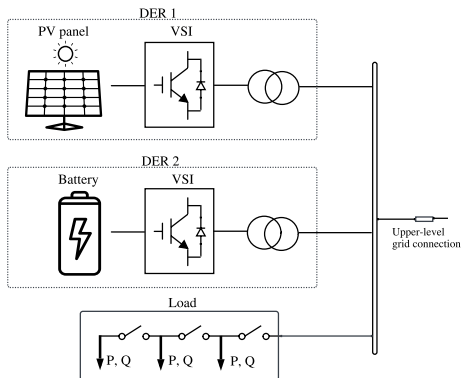


Figure: Inverters in a microgrid.

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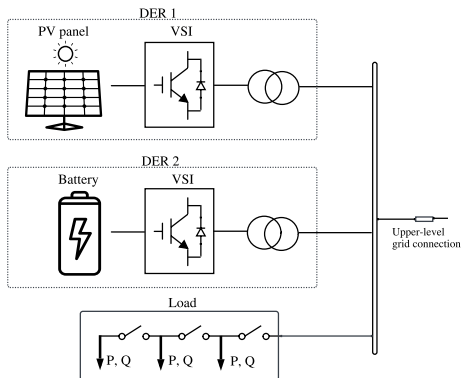


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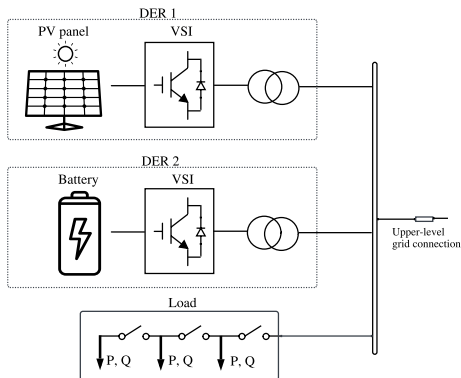


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Contributions

- We propose a non-cooperative game framework that incorporates inverter dynamics
- We learn a state-space representation of the inverter dynamics
- Our control scheme enables a microgrid to provide regulation services to support the upper-level grid
- We show the cost effectiveness and time-domain performance of our proposed control scheme compared with droop control and proportional-integral (PI) control.

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Problem formulation

Problem we want to solve

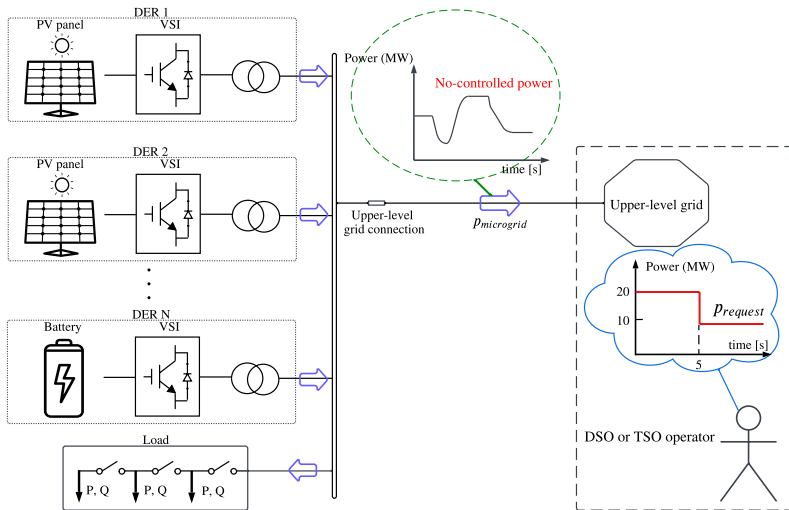


Figure: Need for controlling DERs in support to upper-level grid operation.

Control scheme design

Description of our control scheme (cont.)

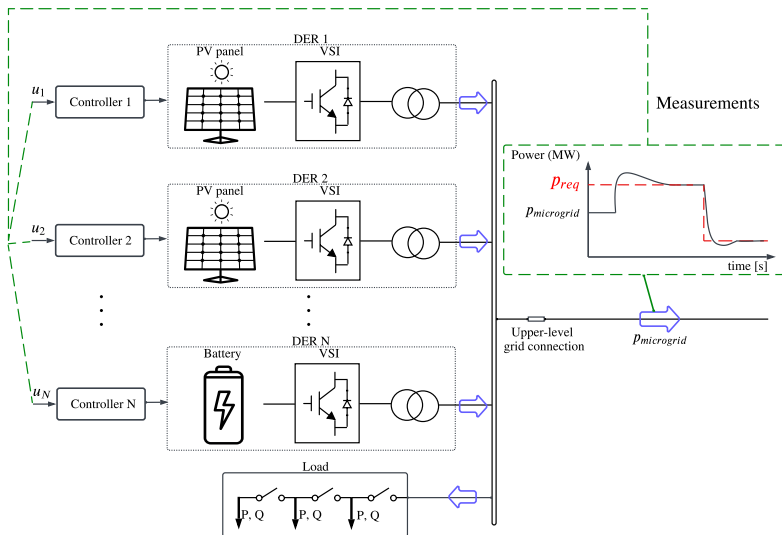


Figure: Control scheme considerations: (i) DERs are selfish, (ii) grid-following inverters, (iii) load perturbations, (iv) $p_{microgrid} \rightarrow p_{req}$

Challenges:

- The state-space representation of each DER is needed
- Deriving exact system dynamics for each DER may come with difficulties:
 - Privacy concerns
 - Multiple control loops with high computational complexity
 - Scalability issues for high number of inverters

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Solution we propose:

Learned inverter dynamics

The dynamics of each inverter are modeled through System Identification (SI). This method identifies the transfer function of a dynamical system from observed input-output data.

Learned inverter dynamics

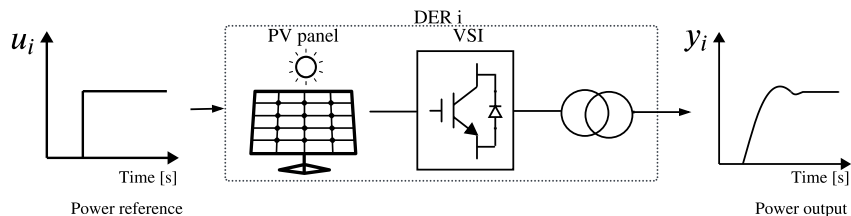


Figure: System Identification extracts dynamics for an inverter-interfaced DER

Using System Identification, the matrices A_i , B_i , C_i and D_i of the state-space representation of DER i is

$$\dot{x}_i = A_i x_i + B_i u_i \quad (1)$$

$$y_i = C_i x_i, \quad (2)$$

where $A_i \in \mathbb{R}^{d \times d}$, $B_i \in \mathbb{R}^{d \times 1}$, $C_i \in \mathbb{R}^{1 \times d}$

Cluster of DERs as a Virtual Power Plant (VPP)

The state-space representation of the VPP that groups “N” DERs is

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_N \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} B_1 & & \\ & \ddots & \\ & & B_N \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} \quad (3)$$

$$y = \begin{bmatrix} C_1 & \dots & C_N \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}, \quad (4)$$

In compact form,

$$\dot{x} = Ax + Bu \quad (5)$$

$$y = Cx, \quad (6)$$

- State of the VPP: $x = [x_1 \ \dots \ x_N]^T \in \mathbb{R}^{N \cdot d}$
- Control action of DER i : u_i
- Power output of the VPP: y

Modeling of Regulation service

Considerations:

- Power reference of the regulation service: $p_{\text{req}}(t) \in \mathbb{R}$,
- MG's power delivered to the upper grid $y(t) - d(t) \rightarrow p_{\text{req}}(t)$,
- To comply with the regulation service, we use a compensator (i.e., achieving zero steady-state error) with state w , output v , matrices H , G and D .

Using the deviation of the states, the resulting augmented dynamics for the VPP is:

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{w}} \end{bmatrix} = \bar{A} \begin{bmatrix} \tilde{x} \\ \tilde{w} \end{bmatrix} + [\bar{B}_1 \quad \dots \quad \bar{B}_N] \tilde{u} \quad (7)$$

$$\begin{bmatrix} \tilde{y} \\ \tilde{v} \end{bmatrix} = \bar{C} \begin{bmatrix} \tilde{x} \\ \tilde{w} \end{bmatrix}, \quad (8)$$

where $\bar{A} = \begin{bmatrix} A & 0 \\ -GC & H \end{bmatrix}$, $\bar{B}_i = [0 \quad \dots \quad B_i \quad \dots \quad 0]^T$, and $\bar{C} = \begin{bmatrix} C & 0 \\ 0 & D \end{bmatrix}$.

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Non-cooperative linear quadratic differential game for DER coordination

Each DER seeks to minimize its individual cost $J_i(\tilde{x}_0, \tilde{w}_0, \tilde{u})$ during the power regulation service. The cost is given by

$$J_i(\tilde{x}_0, \tilde{w}_0, \tilde{u}) = \int_{t_0}^{\infty} \left\{ \begin{bmatrix} \tilde{x} \\ \tilde{w} \end{bmatrix}^T Q_i \begin{bmatrix} \tilde{x} \\ \tilde{w} \end{bmatrix} + \tilde{u}_i^T R_i \tilde{u}_i \right\} dt, \quad (9)$$

- where $Q_i = Q_i^T \geq 0$ and $R_i \geq 0$

Subject to:

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{w}} \end{bmatrix} = \bar{A} \begin{bmatrix} \tilde{x} \\ \tilde{w} \end{bmatrix} + [\bar{B}_1 \quad \dots \quad \bar{B}_N] \tilde{u} \quad (10)$$

$$\begin{bmatrix} \tilde{y} \\ \tilde{v} \end{bmatrix} = \bar{C} \begin{bmatrix} \tilde{x} \\ \tilde{w} \end{bmatrix}, \quad (11)$$

Nash equilibrium strategy for DERs

Each DER employs a linear feedback strategy given by

$$\tilde{u}_i = \begin{bmatrix} K_i & F_i \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{w} \end{bmatrix}, \quad (12)$$

We determine the set of admissible strategies $\{u_1, \dots, u_N\}$ of the form (12) using the concept of Nash equilibrium

$$J_i(\tilde{x}_0, \tilde{w}_0, [\tilde{u}_1^*, \dots, \tilde{u}_i^*, \dots, \tilde{u}_N^*]) \leq J_i(\tilde{x}_0, \tilde{w}_0, [\tilde{u}_1^*, \dots, \tilde{u}_i, \dots, \tilde{u}_N^*]), \quad (13)$$

for $i = \{1, 2, \dots, N\}$.

Nash equilibrium strategy for DERs (cont.)

$$u_i^* = -R_i^{-1} \bar{B}_i P_i \begin{bmatrix} \tilde{x} \\ \tilde{w} \end{bmatrix} \quad (14)$$

for $i = \{1, \dots, N\}$.

The matrices P_i are the symmetric stabilizing solution of the coupled Algebraic Riccati equations:

$$\left(\bar{A} - \sum_{j \neq i}^N S_j P_j \right)^\top P_i + P_i \left(\bar{A} - \sum_{j \neq i}^N S_j P_j \right) - P_i S_i P_i + Q_i = 0 \quad (15)$$

for $i = \{1, \dots, N\}$, where $S_i = \bar{B}_i R_i^{-1} \bar{B}_i^\top$.

We use an iterative algorithm⁶ to obtain P_i .

⁶Jacob Engwerda. "Algorithms for computing Nash equilibria in deterministic LQ games". In: *Computational Management Science* 4.2 (Apr. 2007), pp. 113–140. ISSN: 1619-6988.

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State estimation of DERs

- We need state variables information to calculate control action u_i
- In our setup there are no state variable measurements (as they are internal states)
- We use **Loop Transfer Recovery (LTR)** to estimate the states (variant of the Kalman Filter)
- LTR is robust to parameter perturbations ΔA_i , ΔB_i , ΔC_i from the learned DER dynamics

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Control scheme

In summary, the state-feedback control and LTRs are placed in the MG as follows:

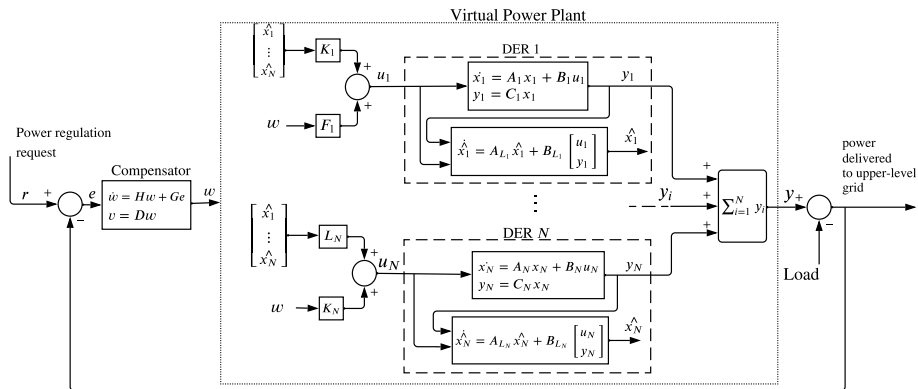


Figure: Control scheme for a microgrid (MG) to provide power regulation service in support to the upper-level grid.

Simulations and Results

Validation of learned dynamics and control scheme

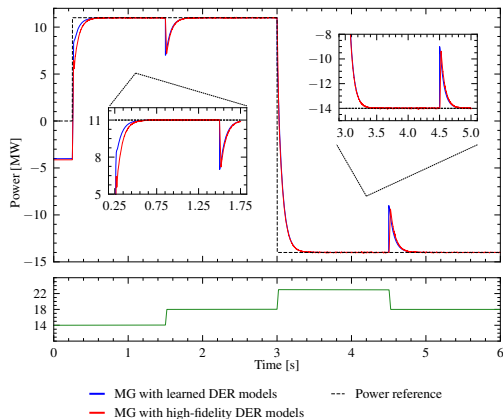


Figure: Top panel: Microgrid (MG)'s power output with learned DER models and MG's power output with high-fidelity DER models. Bottom panel: MG's load during the regulation service.

Case study

The four scenarios correspond to 10-kV MGs with different numbers of DERs:

- 1 PV system and 1 BESS
- 1 PV system and 2 BESS
- 3 PV systems and 3 BESS
- 4 PV systems and 6 BESS

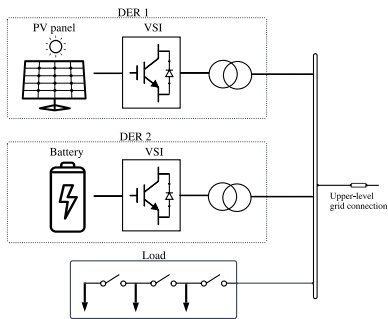


Figure: MG in scenario 1

Comparison of the proposed control scheme against droop and PI control using high-fidelity dynamics

Benchmark of the proposed control scheme against droop controller and PI controller

The cost savings of DER i : $\frac{\text{Individual cost}_{\text{using droop/PI}}}{\text{Individual cost}_{\text{proposed}}}$

Table: Cost savings of each DER when using our proposed control scheme for all 4 scenarios using Sandia's high-fidelity inverter models

Savings relative to:	DERs									
	1	2	3	4	5	6	7	8	9	10
Droop	28.3	34.2								
PI	1.3	1.5								
Droop	100	116	123							
PI	3.6	4.1	4.3							
Droop	209	185	204	189	196	171				
PI	9.3	8.3	9.1	8.5	8.8	7.7				
Droop	48.5	50.9	54.2	53.3	51.2	37.0	50.5	51.3	48.7	46.8
PI	7.3	7.6	8.1	8.0	7.7	5.7	7.6	7.7	7.3	7.1

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PI	1.3	1.5								
Droop	100	116	123							
PI	3.6	4.1	4.3							
Droop	209	185	204	189	196	171				
PI	9.3	8.3	9.1	8.5	8.8	7.7				
Droop	48.5	50.9	54.2	53.3	51.2	37.0	50.5	51.3	48.7	46.8
PI	7.3	7.6	8.1	8.0	7.7	5.7	7.6	7.7	7.3	7.1

Benchmark in time-domain performance

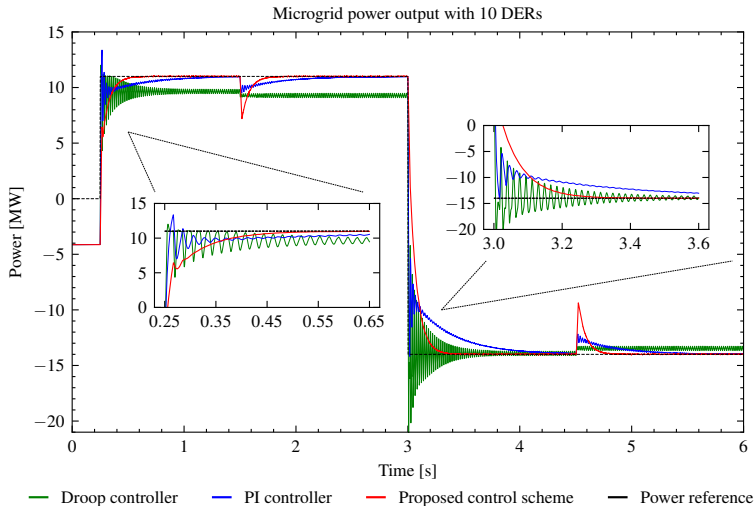


Figure: Pros: Faster settling times, no oscillations, and zero steady-state error. Cons: Slightly higher overshoot.

Conclusions and Future Directions

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Future Directions

- Diverse set of inverter manufacturers and different operational points for inverters lead to heterogeneous and time-varying dynamics.
- Challenges: how to derive or represent aggregate dynamics, tractability issues, etc.
- **Learning inverter dynamics** is a promising field of research.
- How to **guarantee stabilizing solutions** for ancillary services from clusters of DERs considering communication delays in state estimation, or partial information.
- In a broader setup, where the transmission network is supported by the distribution network to provide frequency support, closed-loop stabilizing control becomes essential, and much more challenging.
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Thank you!

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Supplemental information

Modeling of inverters

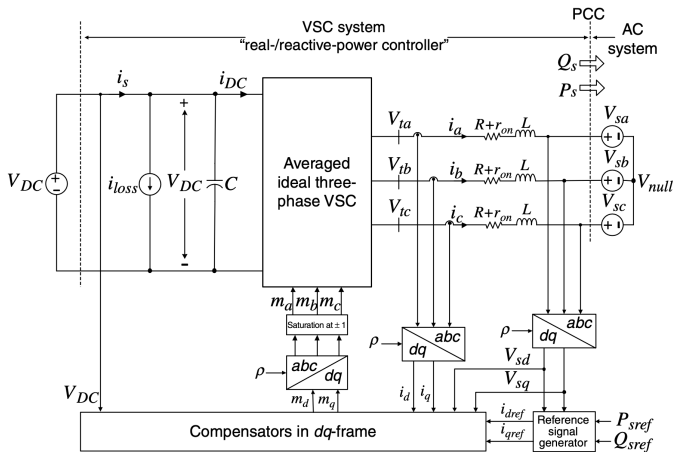


Figure: A grid-following inverter with compensators in dq -frame able to track P_{sref} , Q_{sref} .

Learned state-space representation of DERs

By System Identification, we obtain the following state-space models

Table: DERs learned state-space representations

Parameter	PV system	BESS
A	$\begin{bmatrix} -263.094 & -2.955 \cdot 10^4 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -258.087 & -3.041 \cdot 10^4 \\ 1 & 0 \end{bmatrix}$
B	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
C	$[1.589 \quad 2.945 \cdot 10^4]$	$[9.712 \quad 3.039 \cdot 10^4]$

As system parameters may vary in practice, we intentionally introduce perturbations to the LTR estimator of each DER across all scenarios.

Table: Parameter perturbations introduced to the LTR estimators

Parameter	PV system	BESS
ΔA	$\begin{bmatrix} -20 & 1000 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -100 & 1000 \\ 1 & 0 \end{bmatrix}$
ΔB	$\begin{bmatrix} -0.1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -0.1 \\ 0 \end{bmatrix}$
ΔC	$[0.1 \quad 0.1]$	$[0.1 \quad 0.1]$

Learned voltage source inverter dynamics

In System Identification (SI)⁸, we provide input-output data set

$$f(t) = [-y(t-1) \dots -y(t-n) \ u(t-1) \dots u(t-m)]^\top. \quad (16)$$

⁸L. Ljung. *System Identification: Theory for the User*. Prentice Hall information and system sciences series. Prentice Hall PTR, 1999. ISBN: 9780136566953.

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A candidate dynamical model is proposed, e.g.,

$$y(t) - a_1 y(t-1) + \dots + a_n y(t-n) = b_1 u(t-1) + \dots + b_m u(t-m), \quad (17)$$

$$\hat{y}(t|\alpha) = f(t)^\top \alpha \quad (18)$$

where: $\alpha = [a_1 \dots a_n \ b_1 \dots b_n]$.

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where: $\alpha = [a_1 \dots a_n \ b_1 \dots b_m]$.

The idea is to find α by non-linear least squares methods

$$\hat{\alpha} = \underset{\alpha}{\operatorname{argmin}} \frac{1}{N} \sum_{t=1}^N [y(t) - \hat{y}(t|\theta)]^2 \quad (19)$$

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Modeling of Regulation service

Considerations:

- Power reference of the regulation service: $p_{\text{req}}(t) \in \mathbb{R}$,
- MG's power delivered to the upper grid $y(t) - d(t) \rightarrow p_{\text{req}}(t)$,
- Equivalently, $y(t) \rightarrow r(t) = p_{\text{req}}(t) + d(t)$

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To comply with the regulation service, we propose a compensator

$$\dot{w} = Hw + Ge \quad (20)$$

$$v = Dw, \quad (21)$$

where $w(t), v(t) \in \mathbb{R}$, and $e(t)$ represents the tracking error

$$e(t) := r(t) - y(t) = r(t) - Cx(t). \quad (22)$$

Dynamics of the VPP and compensator

We group the dynamics of the VPP and compensator in the following augmented state-space representation:

$$\begin{bmatrix} \dot{x} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -GC & H \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} + [\bar{B}_1 \quad \dots \quad \bar{B}_N] u + \begin{bmatrix} 0 \\ G \end{bmatrix} r \quad (23)$$

$$\begin{bmatrix} y \\ v \end{bmatrix} = \begin{bmatrix} C & 0 \\ 0 & D \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix}, \quad (24)$$

- State of the VPP: $x \in \mathbb{R}^{N \cdot d}$
- State of the compensator: $w \in \mathbb{R}$
- Control action for the VPP: $u \in \mathbb{R}^N$
- Power reference: $r \in \mathbb{R}$
- Power output of the VPP: $y \in \mathbb{R}$
- Output of the compensator: $v \in \mathbb{R}$

Deviation form of the augmented system

One issue with the augmented system is the presence of \mathbf{r} in

$$\begin{bmatrix} \dot{x} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -GC & H \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} + [\bar{B}_1 \quad \dots \quad \bar{B}_N] u + \begin{bmatrix} 0 \\ G \end{bmatrix} \mathbf{r} \quad (25)$$

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To remove \mathbf{r} , we redefine the augmented system with deviation states⁹:

$$\tilde{x}(t) = x(t) - x_{ss} \quad (27)$$

$$\tilde{w}(t) = w(t) - w_{ss}, \quad (28)$$

where x_{ss} and w_{ss} are the states achieved when the tracking error e becomes zero.

⁹Frank L. Lewis, Draguna Vrabeie, and Vassilis L. Syrmos. *Optimal Control*. John Wiley & Sons, Ltd, 2012. ISBN: 9781118122631.

Deviation form of the augmented system (cont.)

With the deviation states, the augmented system is:

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{w}} \end{bmatrix} = \bar{A} \begin{bmatrix} \tilde{x} \\ \tilde{w} \end{bmatrix} + [\bar{B}_1 \quad \dots \quad \bar{B}_N] \tilde{u} \quad (29)$$

$$\begin{bmatrix} \tilde{y} \\ \tilde{v} \end{bmatrix} = \bar{C} \begin{bmatrix} \tilde{x} \\ \tilde{w} \end{bmatrix}, \quad (30)$$

where $\bar{A} = \begin{bmatrix} A & 0 \\ -GC & H \end{bmatrix}$, $\bar{B}_i = [0 \quad \dots \quad B_i \quad \dots \quad 0]^T$, and $\bar{C} = \begin{bmatrix} C & 0 \\ 0 & D \end{bmatrix}$.

Equivalence of tracker problem and regulator problem

A tracker problem for the augmented system is actually equivalent to a regulator problem for its corresponding deviation system^a.

^aFrank L. Lewis, Draguna Vrabeie, and Vassilis L. Syrmos. *Optimal Control*. John Wiley & Sons, Ltd, 2012. ISBN: 9781118122631.

Validation of the control scheme

Validation of the control scheme

- First, we determine the controllers for each DER following the non-cooperative game approach. In this step, learned state-space representations of the DERs are used.
- Second, in each of the 4 scenarios we do two separate implementations:
 - the controllers in the grid with learned state-space models.
 - the controllers in the grid with high-fidelity DER models (realistic case).

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Validation of control scheme

We compare objective functions and time-domain performance of:
(i) grid with learned state-space models vs (ii) grid with high-fidelity DER models

Validation of control scheme (analysis of cost)

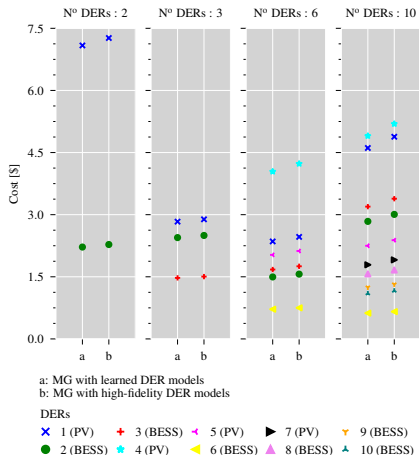


Figure: Optimal individual costs for each DER in case: (a) the microgrid (MG) with learned DER models and (b) the MG with high-fidelity DER models for all four scenarios.

Validation of control scheme (analysis of time-domain performance)

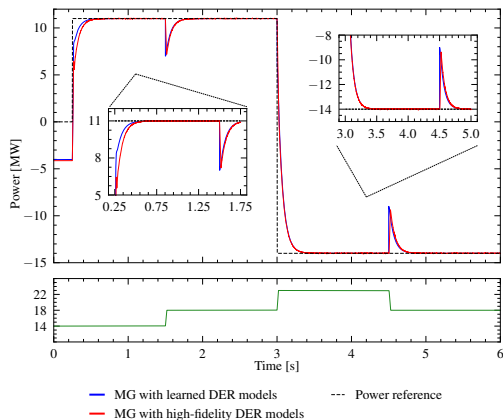


Figure: Top panel: Microgrid (MG)'s power output with learned DER models and MG's power output with high-fidelity DER models. Bottom panel: MG's load during the regulation service.

Benchmark in time-domain performance

With the proposed control scheme, the VPP has: (i) no oscillations, (ii) faster settling times, (iii) achieves almost zero steady-state error.

Table: MG's performance for three control schemes in all four scenarios

DERs	Control	Overshoot (%)	Settling time (s)	steady-state error (%)	Damping (ζ)
2	Droop	-65.5	0.09	37.67	0.12
	PI	-56.02	0.69	1.2	0.12
	Proposed	-35.67	0.42	0.01	1
3	Droop	-61.36	0.1	28.88	0.25
	PI	-17.74	0.61	0.24	0.13
	Proposed	-35.09	0.26	0	1
6	Droop	-37.37	0.07	28.96	0.06
	PI	21.61	0.63	0.69	0.10
	Proposed	-33.93	0.21	0	1
10	Droop	-54	0.19	15.7	0.53
	PI	22.15	0.68	0.66	0.09
	Proposed	-34.6	0.23	0.02	1