

Resource-Aware Network Optimization in Autonomous Energy Grids

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Joint w/: Priyank Srivastava
Guido Cavraro



Complex Cyberphysical Systems

We live in an **interconnected** world



environmental sensing



industrial internet



energy grids



connected vehicles



IoT



Google Loon

Tight coupling between physical and cyber processes:

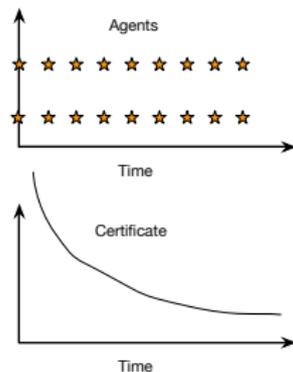
convergence of control, communication, computing, storage, sampling, signal processing, estimation, interaction with humans

Need for efficient use of available resources

Resource-Aware Control and Coordination

Continuous or **periodic** implementation paradigm

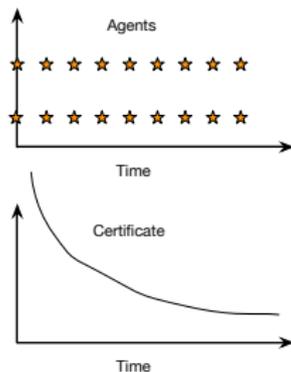
- costly-to-implement **synchronization** for information sharing, processing, decision making
- **'passive'** asynchronism, fixed agent time schedules
- **inefficient** implementations for processor usage, communication bandwidth, energy



Resource-Aware Control and Coordination

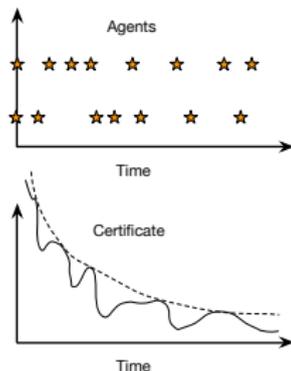
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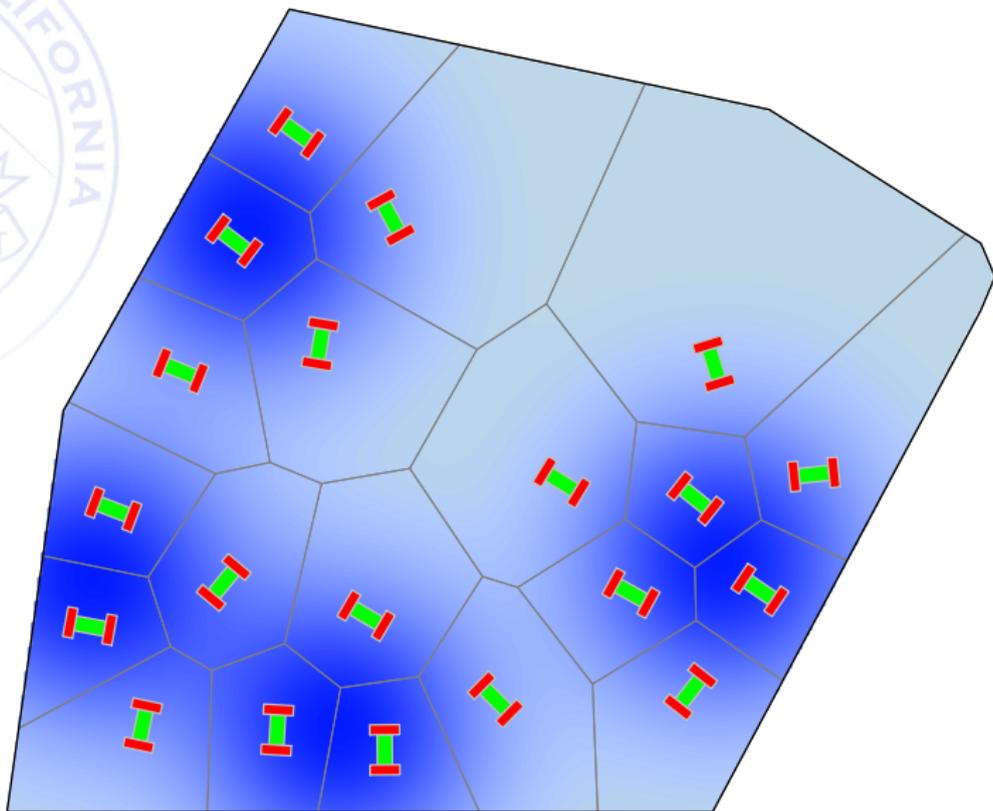


Opportunistic state-triggered paradigm

- **trade-offs**: comp, comm, sensing, control
- identify criteria to autonomously trigger actions based on task – 'active' asynchronism
- **efficient** implementations, incorporates uncertainty



A Movie is Worth a Thousand Words



How to Decide When to Update?

Simplified setup: system $\dot{x} = f(x, u)$ on \mathbb{R}^n with stabilization via

• **controller:** $k : \mathbb{R}^n \rightarrow \mathbb{R}^m$

• **certificate:** Lyapunov function $V : \mathbb{R}^n \rightarrow \mathbb{R}$

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Synthesis for $\dot{x} = f(x, k(\bar{x}))$? (w/ \bar{x} constructed from sampled information of x)

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By ensuring $\dot{V} = \nabla V(x) \cdot f(x, k(\bar{x})) < 0$

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$$\begin{aligned} \dot{V} &= \nabla V(x) \cdot f(x, k(\bar{x})) \\ &= \nabla V(x) \cdot f(x, k(x)) + \nabla V(x) \cdot (f(x, k(\bar{x})) - f(x, k(x))) \end{aligned}$$

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$$\begin{aligned} \dot{V} &= \nabla V(x) \cdot f(x, k(\bar{x})) \\ &\leq \nabla V(x) \cdot f(x, k(x)) + h(x) \|\bar{x} - x\| \leq 0 \end{aligned}$$

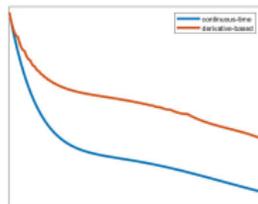
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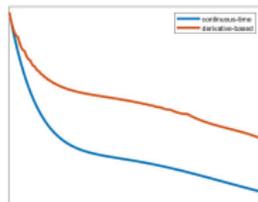
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- **feasibility:** rule out accumulation of trigger times



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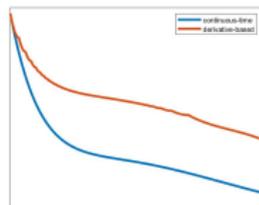
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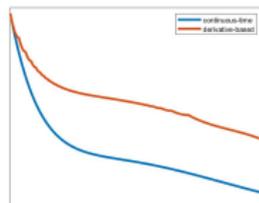
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- **trigger evaluation:** computable with available information



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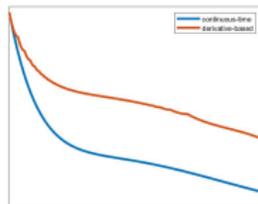
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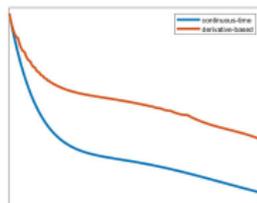
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- what is **resource** to be aware of?



What is the 'Resource' to Be Aware of?

Started off with **actuator** updates for **stabilization**, expanded to whole plant-sensor-actuator-controller model

P. Tabuada. Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic Control*, 52(9):1680–1685, 2007

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Increasingly richer notions of **'resource'**

- communication among individual agents to achieve collective task
- understanding stabilization under information constraints
- actuator updates for safety-critical systems
- costly recomputation of optimal policy
- re-sampling system state in accelerated optimization
- requesting information from a human

Today: Resource-Aware Network Optimization

Specific challenges:

- **feasibility:** emergence of Zeno behavior b/c of availability of partial information to agents
- **trigger evaluation:** solution of optimization problem is not known a priori!
- **trigger evaluation:** criterion for individual agents computable with locally available information



Agent-supervisor coordination strategy

decentralized, opportunistic, guarantees anytime feasibility and asymptotic convergence to optimizer

Problem Formulation

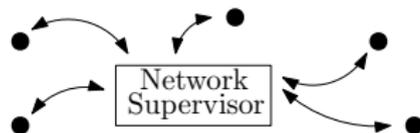

$$\min_{x \in \mathcal{X}} \sum_{i=1}^n f_i(x_i) + g(x)$$

Problem data

- f_i : **local** cost function of agent i , depends on its own state
- g : **coupling** cost function, depends on network state
- $\mathcal{X} = \prod_{i=1}^n \mathcal{X}_i$: separable constraint set

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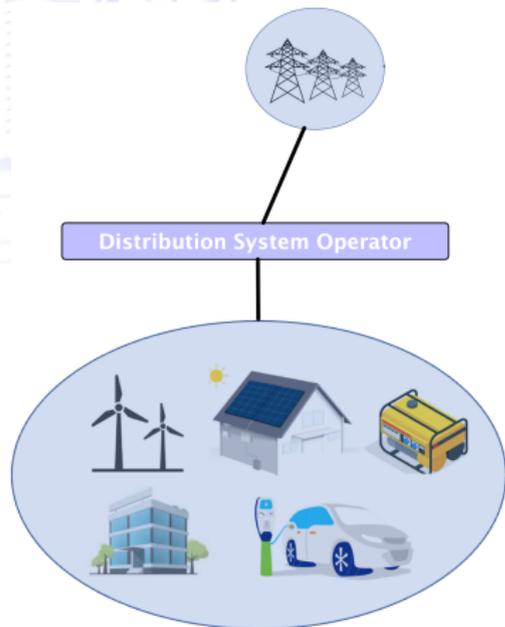


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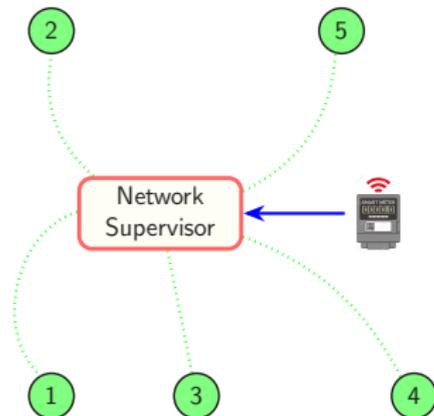
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Communication: agents rely on supervisor to obtain information about coupling cost

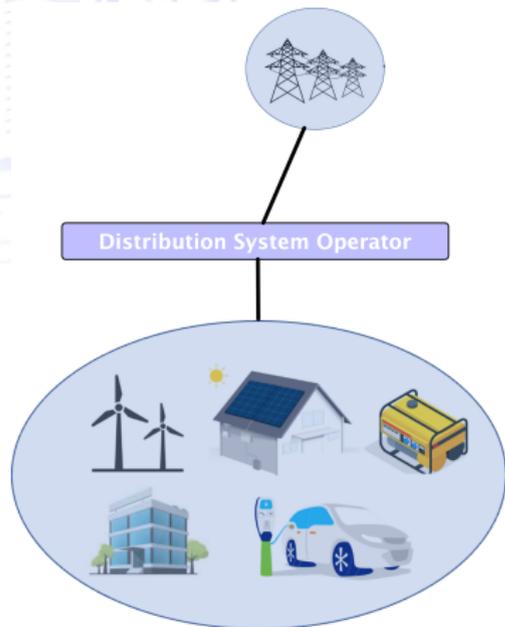
Prosumer-Based Distribution Network



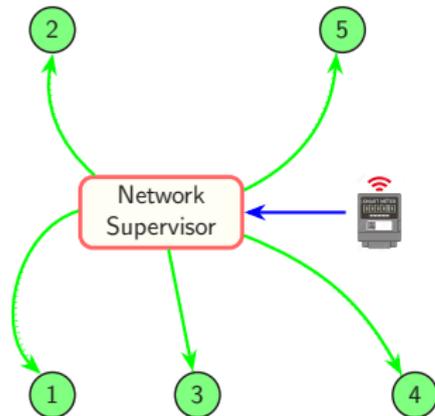
- **prosumers:** internal generation cost
- **substation:** power exchange cost w/ grid



Prosumer-Based Distribution Network



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Event-Triggered Agent-Supervisor Coordination

Network optimization

$$\min_{x \in \mathcal{X}} \sum_{i=1}^n f_i(x_i) + g(x)$$

- Without coupling cost g , each agent simply would solve $\min_{x_i \in \mathcal{X}_i} f_i(x_i)$
- Presence of g couples agents' decisions, requiring continuous or periodic agent-supervisor communication

Goal: endow agents with locally evaluable criterion to **opportunistically** decide when to request information

Unconstrained Case

Gradient descent

$$\dot{x}_i = - \underbrace{\nabla_i f_i(x_i)}_{\text{agent}} - \underbrace{\nabla_i g(x)}_{\text{supervisor}}, \quad i \in \{1, \dots, n\}$$

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Requires supervisor to provide information continuously!

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Opportunistic implementation

$$\dot{x}_i = - \underbrace{\nabla_i f_i(x_i)}_{\text{agent}} - \underbrace{\nabla_i g(x(t_k))}_{\text{supervisor}}$$

Requires supervisor to provide information at triggering times $\{t_k\}_{k=0}^{\infty}$

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How to determine $\{t_k\}_{k=0}^{\infty}$ in a decentralized fashion and ensure convergence?

Decentralized Event-Triggered Coordination

- 1 each agent $i \in \{1, \dots, n\}$ evaluates

$$t_{k+1}^i = \min \{t > t_k \mid L_g |x_i - x_i(t_k)| = \sigma |\nabla_i f_i(x_i) + \nabla_i g(x(t_k))|\}$$

(L_g Lipschitz constant of ∇g and $\sigma \in (0, 1)$ design parameter)

- 2 whoever finds smallest time, determines triggering time

$$t_{k+1} = \min_{i \in \{1, \dots, n\}} t_{k+1}^i$$

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Objective Function is Monotonically Decreasing

$V(x) = f(x) + g(x) - f(x^*) - g(x^*)$ is monotonically decreasing along $\cup_{k=0}^{\infty} [t_k, t_{k+1}]$

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Proof follows from $\dot{V} \leq -\|\dot{x}\|^2 \left(1 - \frac{\|\nabla g(x) - \nabla g(x(t_k))\|}{\|\dot{x}\|}\right)$

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Minimum Inter-Event Time and Convergence to Optimizer

There exists $\tau > 0$ such that $t_{k+1} - t_k \geq \tau$ for all k , and any trajectory of opportunistic implementation converges to optimizer

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Proof relies on lower-bounding time it takes state estimation error to reach triggering threshold

Constrained Case

Optimization via Continuous Projected Dynamics

Continuous projected dynamics

$$\dot{x} = \Pi_{\mathcal{X}}(x - \lambda(\nabla f(x) + \nabla g(x))) - x$$

where $\lambda > 0$ is design parameter

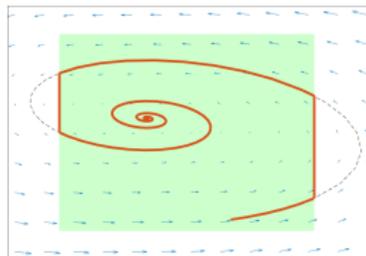
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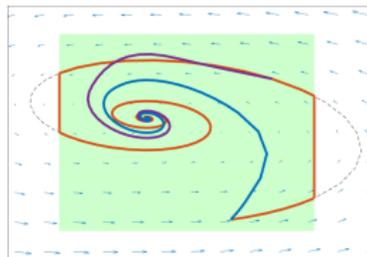
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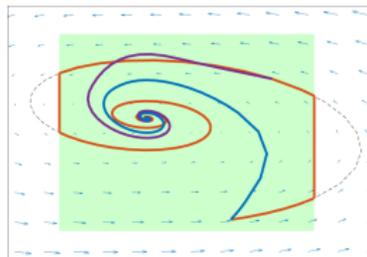
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If $f + g$ has Lipschitz gradient,

- feasible set is forward invariant and attractive
- dynamics converges to optimizer

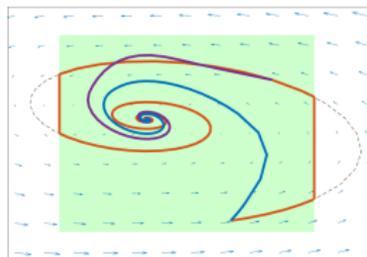
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Opportunistic implementation

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$$t_{k+1}^i = \min \left\{ t > t_k \mid \lambda \bar{L}_g |x_i - x_i^k| = \sigma |\Pi_{\mathcal{X}_i}(x_i - \lambda(\nabla_{x_i} f_i(x_i) + \nabla_{x_i} g(x(t_k)))) - x_i| \right\}$$

(\bar{L}_g Lipschitz constant of ∇g over \mathcal{X} and $\sigma \in (0, 1)$ design parameter)

- 2 whoever finds smallest time, determines triggering time

$$t_{k+1} = \min_{i \in \{1, \dots, n\}} t_{k+1}^i$$

- 3 supervisor provides information to compute $\nabla_i g(x(t_{k+1}))$ to each $i \in \{1, \dots, n\}$

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Objective Function is Monotonically Decreasing

$V(x) = f(x) + g(x) - f(x^*) - g(x^*)$ is monotonically decreasing along $\cup_{k=0}^{\infty} [t_k, t_{k+1}]$

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Minimum Inter-Event Time and Convergence to Optimizer

For $\lambda < 1/H$, there exists $\tau > 0$ such that $t_{k+1} - t_k \geq \tau$ for all k , and any trajectory of opportunistic implementation converges to optimizer

$$H = \max_{i \in \{1, \dots, n\}} \max_{x_i \in \mathcal{X}_i} \nabla_{x_i}^2 f_i(x_i)$$

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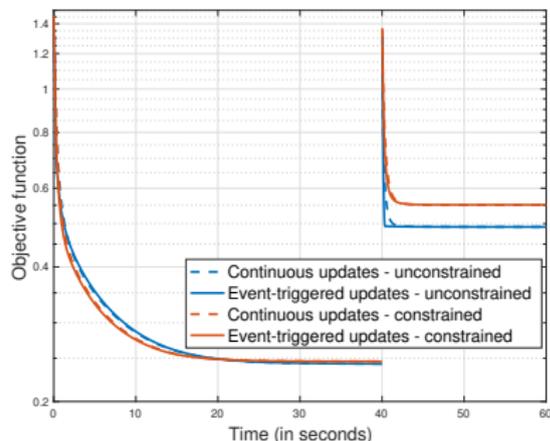
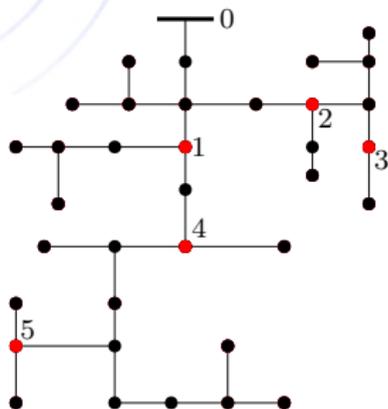
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Proof uses nonsmooth analysis to lower-bound time it takes state estimation error to reach threshold

Simulations on IEEE 37-Bus Test Feeder

Minimizing generation cost in prosumer-based distribution network

$$\min_{x \in \mathcal{X}} \sum_{i=1}^5 f_i(x_i) + f_0(-\mathbf{1}^\top x + c)$$

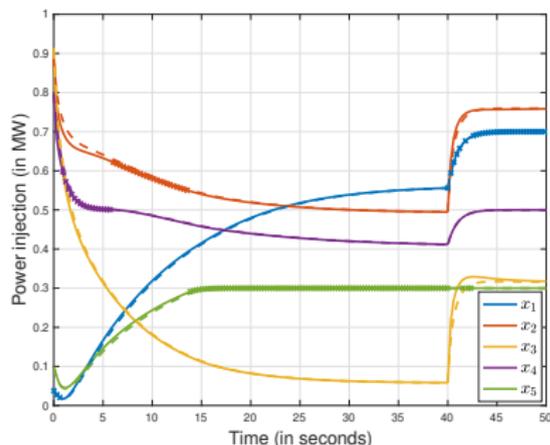
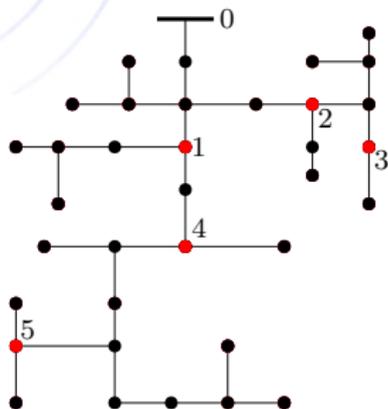


red nodes are generators, node 0 is supervisor

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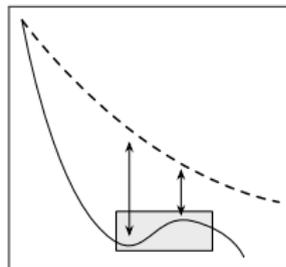
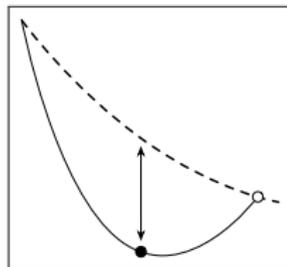
Summary

Resource-aware control and coordination

rich paradigm for real-time implementation of cyberphysical systems

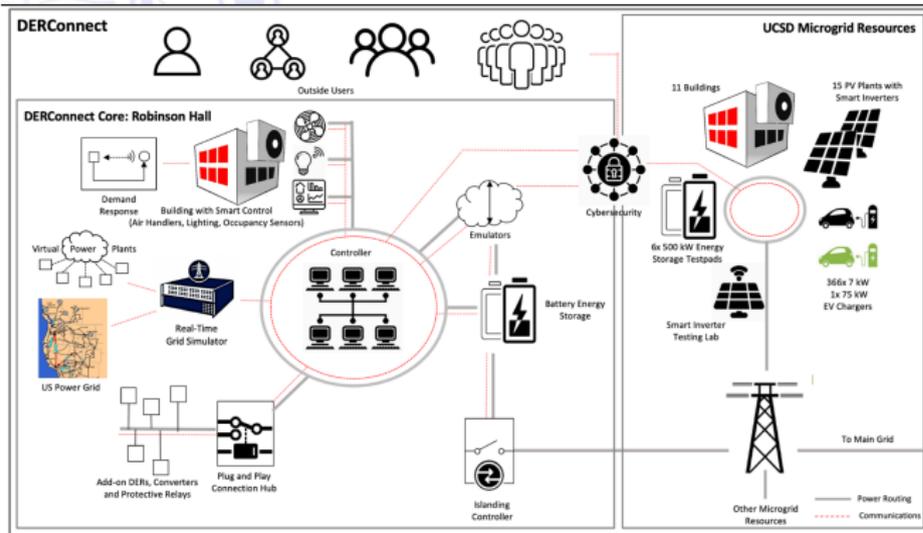
Outlook

- trigger design: performance vs efficiency vs implementability
- trigger evaluation: synthesis of distributed triggers, adaptive budgets
- distributed asynchronism
- resource understood broadly
- elaborate uses of sampling information



DERConnect

NSF User Facility for Control of Distributed Energy Resources



Testbed for distributed controls

- 2,5K controlled devices
- 30K metered devices
- 2M simulated nodes
- remote access nationally

Beyond: cybersecurity, building operations, human-cyber-physical systems

Help Us Shape It

Call for **feedback** from research community:

- types of tests you envision?
- capabilities in terms of communication, sensing, control, and actuation?
- ability to determine test conditions¶meters, templates, tools, libraries
- resolution, granularity, data access and availability during&after tests

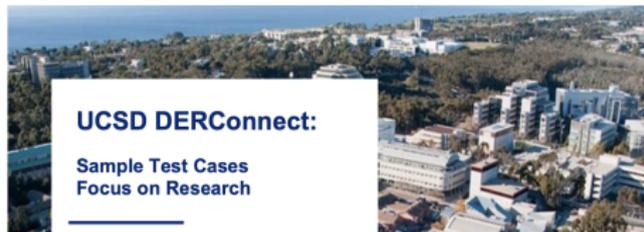
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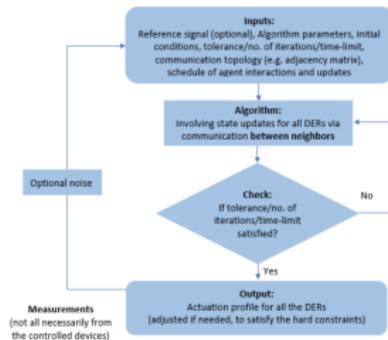
Opportunity to shape capabilities&functionality of DERConnect

Current **live document** outlines 4 high-level skeletons of envisioned test cases



Skeleton 1 – Fully Distributed Actuation

Skeleton 1 describes fully distributed algorithms in which individual nodes communicate with neighbors to solve an optimization or control problem. These algorithms typically involve a mix of local calculations and updates between neighboring DERs. Examples are ARPA-e teams 4 and 6.



<https://sites.google.com/ucsd.edu/derconnect>